Tree diagrams

Construct and read a probability tree for two consecutive events; multiply along branches for a combined probability.

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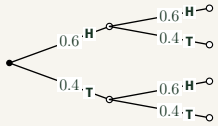
Show your method

Branches leaving the same point sum to 1. Multiply the probabilities along the branches leading to an outcome. With replacement, the second-stage probabilities are unchanged; without replacement, they change.

Worked example A biased coin has $P(\text{Heads}) = 0.6$. Tossed twice, independently. Find $P(\text{HH})$.



Multiply along the branches: $P(\text{HH}) = 0.6 \times 0.6$.
$= 0.36$

1. What is $P(\text{HT})$?

Solution
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. What is $P(\text{TT})$?

Solution
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

3. List all four outcomes of tossing a coin twice.

Solution
HH, HT, TH, TT

4. Find $P(\text{Burger then Juice})$.

Solution
$0.6 \times 0.7 = 0.42$

5. Find $P(\text{Pizza then Cola})$.

Solution
$0.4 \times 0.3 = 0.12$

6. What must the probabilities of branches leaving the same point always sum to?

Solution
1

7. A bag has 3 red, 2 blue. Drawn, replaced, drawn again. Find $P(\text{red})$ on the 2nd draw.

Solution
$\frac{3}{5}$ (unchanged)

8. Using the same tree, find $P(\text{blue then blue})$.

Solution
$\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

9. A tree's first branch is $P(\text{Win}) = 0.3$. Find $P(\text{Lose})$.

Solution
$1 - 0.3 = 0.7$

Tree diagrams (continued)

Compare with-replacement and without-replacement trees; combine leaf probabilities.

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10. Without replacement, given the 1st is red, find $P(\text{red})$ on the 2nd draw.

Solution

$\frac{2}{4} = \frac{1}{2}$

11. Using the same without-replacement tree, find $P(\text{red then red})$.

Solution

$\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$

12. Explain why without-replacement probabilities change on the 2nd draw but with-replacement ones don't.

Solution

Removing a counter changes what's left in the bag; replacing it restores the original total each time.

13. Find $P(\text{at least one Head})$ in two tosses.

Solution

$1 - P(\text{TT}) = 1 - \frac{1}{4} = \frac{3}{4}$

14. Using the without-replacement tree, find $P(\text{one red, one blue, either order})$.

Solution

$\frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$

15. A without-replacement tree starts with 10 counters, 4 red. Find $P(\text{1st red})$.

Solution

$\frac{4}{10} = \frac{2}{5}$

16. Following Q15, if the 1st counter drawn was red, how many counters (and how many red) remain for the 2nd draw?

Solution

9 counters total, 3 red remain.

17. A two-branch tree has probabilities 0.45 and blank at the first stage. Find the missing probability.

Solution

$1 - 0.45 = 0.55$

18. A student tosses a coin twice, gets Heads first, and says Tails is now more likely. Explain why this is wrong.

Solution

The tosses are independent – the coin has no memory, so $P(\text{Tails})$ stays $\frac{1}{2}$ regardless of the 1st toss.