Probability assumptions

Explain and critique the assumptions behind theoretical probability: fair devices, equally likely outcomes, and randomness.

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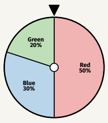
Three core assumptions

Theoretical probability (favourable $\div$ total) only works when a device is FAIR, its outcomes are EQUALLY LIKELY, and the process is RANDOM (uncontrolled). If any assumption fails, you need extra information – the actual proportions, or experimental data – before you can state a probability.

Worked example

Can you say $P(\text{Red}) = \frac{1}{3}$ for this spinner? Explain, and give the correct $P(\text{Red})$.



No – the 3 sectors are not equally sized, so counting sectors doesn't give the probability.
$P(\text{Red}) = 50\% = 0.5$, read directly from its share.

1. What does "fair" mean when we describe a coin or a die?

Solution
Every outcome has an equal chance of occurring – no side or face is favoured over any other.

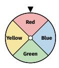
2. This fair die shows Face 4. What is $P(\text{rolling a } 4)$?

Solution
$P(4) = \frac{1}{6}$ (6 equally likely faces).

3. Why must a device be fair before we use $\frac{\text{favourable}}{\text{total}}$ to find a probability?

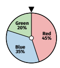
Solution
The fraction method assumes every outcome is equally likely; without fairness, that assumption breaks down.

4. This spinner has 4 equal sectors. Are all four colours equally likely? State $P(\text{Red})$.



Solution
Yes, equal sectors. $P(\text{Red}) = \frac{1}{4}$.

5. Can you use $\frac{1}{3}$ for the largest sector here? What should you use instead?



Solution
No. $P(\text{Red}) = 0.45$.

6. When is it reasonable to treat outcomes as equally likely?

Solution
When the device is symmetric/well-made (e.g. a true die or coin) with no reason to favour one outcome.

7. A coin lands Heads 7 times in 10 tosses. Does this prove it is unfair? Explain.

Solution
No – a fair coin easily produces 7/10 heads from random variation in a small sample.

8. Compare 7 heads in 10 tosses with 700 heads in 1000 tosses. Which is more convincing evidence of bias, and why?

Solution
700/1000: the same 70% in a much larger sample is far less likely to happen by chance if the coin is fair.

9. Why must a probability experiment be RANDOM, not just possible to repeat?

Solution
If the process is controlled or predictable, outcomes aren't chosen fairly, so the probabilities would be invalid.

Probability assumptions (continued)

Explain and critique the assumptions behind theoretical probability: fair devices, equally likely outcomes, and randomness.

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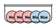
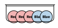
10. 3 red, 2 blue. WITH replacement, does $P(\text{red})$ change next draw?



Solution

No. $P(\text{red}) = \frac{3}{5}$ every draw.

11. WITHOUT replacement, 1st draw is red. Does $P(\text{red})$ change next? Explain.



Solution

Yes: $P(\text{red}) = \frac{2}{4} = \frac{1}{2}$, not $\frac{3}{5}$.

12. Why does replacement matter for the independence assumption?

Solution

Without replacement, removing an item changes the total, so later draws depend on earlier ones – no longer independent.

13. A game advertises a "1 in 4 chance of winning." What must be true about the 4 outcomes for that claim to make sense?

Solution

The 4 outcomes must be equally likely – a fair, random selection among them.

14. A student says "It hasn't rained for 5 days, so rain is now more likely." What mistake are they making?

Solution

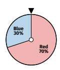
The gambler's fallacy: past independent results do not change the probability of a future one.

15. Give one real-world reason a physical coin or die might not be perfectly fair.

Solution

E.g. an unevenly weighted die, a warped coin, or wear making one side heavier.

16. A carnival game says "each colour has an equal chance." Is this true for this spinner? Justify using the sector sizes.



Solution

No – Red 70%, Blue 30%: unequal.

17. Why might experimental probability be a better guide than theoretical probability for a device you suspect is biased?

Solution

Theory assumes fairness, which may not hold; experimental data reflects the device's real behaviour.

18. Describe one simple experiment to test whether a spinner is biased, and what result would convince you.

Solution

Spin it 100+ times and compare observed percentages to the fair/expected ones; a large, consistent gap is convincing.