

Level 2 algebra review: variation 2 – solutions



Stage 1: manipulate, factorise, and simplify connected expressions.

Name _____

Date _____

Worked solutions

Restrictions refer to the original expressions. Equivalent factored forms are accepted.

1. Simplify the expression.

Solution

$$3x^2 + 3x - 8$$

2. Expand and simplify.

Solution

$$\begin{aligned} 8x - 12 - 3x - 15 \\ = 5x - 27 \end{aligned}$$

3. Expand and simplify.

Solution

$$\begin{aligned} x^2 - 4x - 12 + 3x \\ = x^2 - x - 12 \end{aligned}$$

4. Expand $(3x + 2)^2$.

Solution

$$9x^2 + 12x + 4$$

5. Simplify using positive indices.

Solution

$$4a^4b^3$$

6. Factorise fully.

Solution

$$7p^2q(2p - 3q)$$

7. Factorise.

Solution

$$(x - 4)(x - 9)$$

8. Factorise.

Solution

$$(3x - 2)(2x + 1)$$

9. Factorise.

Solution

$$(5y - 7z)(5y + 7z)$$

10. Simplify the fraction.

Solution

$$\frac{x - 4}{x + 3}$$
$$x \neq -4, -3$$

11. Simplify the fraction.

Solution

$$\frac{3(x - 4)}{x - 2}$$
$$x \neq 0, 2$$

12. Simplify the fraction.

Solution

$$\frac{(2x + 1)(x - 3)}{(2x - 3)(2x + 3)}$$
$$x \neq \pm \frac{3}{2}$$

13. Simplify the product.

Solution

$$\frac{x - 6}{x + 2}$$
$$x \neq -2, 3$$

14. Write as one fraction.

Solution

$$\frac{-x - 7}{(x + 1)(x - 2)}$$
$$x \neq -1, 2$$

15. Simplify the quotient.

Solution

$$\frac{x + 2}{x}$$
$$x \neq -2, 0, 2$$

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Stage 2: solve equations, inequalities, systems, and algebraic arguments.



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Worked solutions

Substitution can be used to check every numerical solution.

16. Solve the equation.

Solution

$$\begin{aligned}5x &= 35 \\ x &= 7\end{aligned}$$

17. Solve the equation.

Solution

$$\begin{aligned}7x &= 37 \\ x &= \frac{37}{7}\end{aligned}$$

18. Solve the equation.

Solution

$$\begin{aligned}8x + 4 - 3x + 6 &= 60 \\ x &= 10\end{aligned}$$

19. Solve the equation.

Solution

$$\begin{aligned}22 - 6x &= 4x + 4 \\ x &= \frac{9}{5}\end{aligned}$$

20. Solve the equation.

Solution

$$\begin{aligned}8 &= 3x + 6 \\ x &= \frac{2}{3}\end{aligned}$$

21. Solve the quadratic.

Solution

$$\begin{aligned}(x - 3)(x - 8) &= 0 \\ x &= 3, 8\end{aligned}$$

22. Solve the quadratic.

Solution

$$\begin{aligned}(3x + 2)(x - 4) &= 0 \\ x &= -\frac{2}{3}, 4\end{aligned}$$

23. Solve the quadratic.

Solution

$$\begin{aligned}x^2 - 11x + 12 &= 0 \\ x &= \frac{11 \pm \sqrt{73}}{2}\end{aligned}$$

24. Solve the rational equation.

Solution

$$\begin{aligned}x + 3 &= 2x - 2 \\ x &= 5\end{aligned}$$

25. Solve the system.

Solution

$$\begin{aligned}7x &= 18 \\ (x, y) &= \left(\frac{18}{7}, \frac{29}{7}\right)\end{aligned}$$

26. Solve the system.

Solution

$$\begin{aligned}23x &= 53 \\ (x, y) &= \left(\frac{53}{23}, \frac{86}{23}\right)\end{aligned}$$

27. Solve the inequality.

Solution

$$\begin{aligned}-5x &> -25 \\ x &< 5\end{aligned}$$

28. Solve the inequality.

Solution

$$\begin{aligned}-11 &\leq 11x \\ x &\geq -1\end{aligned}$$

29. Prove divisibility by 8.

Solution

$$8n + 16 = 8(n + 2)$$

30. Find both pairs.

Solution

$$\begin{aligned}n(n + 7) &= 60 \\ (5, 12) &\text{ or } (-12, -5)\end{aligned}$$

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Stage 3: use fractional indices and rearrange increasingly nested formulae.

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Worked solutions

Positive-index final forms and principal positive quantities are used where appropriate.

31. Simplify.

Solution

$$a^{-5+9} = a^4$$

32. Simplify.

Solution

$$m^{7/3+2/3} = m^3$$

33. Simplify.

Solution

$$3x^3y^{-2} = \frac{3x^3}{y^2}$$

34. Evaluate.

Solution

$$16^{-3/2} = \frac{1}{64}$$

35. Evaluate.

Solution

$$32^{4/5} = 2^4 = 16$$

36. Solve.

Solution

$$x = 16^{3/4} = 8$$

37. Solve.

Solution

$$x^{2/3} = 9$$
$$x = 27$$

38. Simplify.

Solution

$$= (a^{-2}b^3)^2$$
$$= \frac{b^6}{a^4}$$

39. Simplify.

Solution

$$= p^{-1}q^3$$
$$= \frac{q^3}{p}$$

40. Make t the subject.

Solution

$$at^2 + 2ut - 2s = 0$$
$$t = \frac{-u \pm \sqrt{u^2 + 2as}}{a}$$

41. Make h the subject.

Solution

$$h = \frac{V}{\pi r^2}$$

42. Make x the subject.

Solution

$$3xy - 4y = 5x + 2$$
$$x = \frac{4y + 2}{3y - 5}$$

43. Make r the subject.

Solution

$$4r^2 + 2hr - \frac{S}{\pi} = 0$$
$$r = \frac{-h \pm \sqrt{h^2 + 4S/\pi}}{4}$$

44. Make m the subject.

Solution

$$\frac{T^2}{4\pi^2} = \frac{m}{k}$$
$$m = \frac{kT^2}{4\pi^2}$$

45. Make x the subject.

Solution

$$bx - c = \left(\frac{P}{a}\right)^{1/n}$$
$$x = \frac{c + (P/a)^{1/n}}{b}$$

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Stage 4: combine restrictions, powers, roots, and multi-step rearrangement.

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Worked solutions

Exact answers are retained; invalid roots are rejected against the original equation.

46. Make a the subject.

Solution

$$a = \frac{b(k+3)}{5k-2}$$

47. Make b the subject.

Solution

$$b = \left(\frac{Pa^{2/3}}{c} \right)^{2/5}$$

48. Make x the subject.

Solution

$$x = \frac{dy^2 + b}{cy^2 - a}$$

49. Solve the rational equation.

Solution

$$x^2 - 3x + 15 = 0$$

 No real solutions.

50. Solve the rational equation.

Solution

$$2x^2 - 5x - 3 = 0$$

$$x = 3, -\frac{1}{2}$$

51. Simplify the fraction.

Solution

$$\frac{(3x+1)(x-4)}{(3x+2)(2x-1)}$$

$$x \neq -\frac{2}{3}, \frac{1}{2}$$

52. Solve the exponential equation.

Solution

$$3x - 3 = 4x + 2$$

$$x = -5$$

53. Solve the exponential equation.

Solution

$$6x = 2x + 2$$

$$x = \frac{1}{2}$$

54. Solve the radical equation.

Solution

$$x = 2 \pm \sqrt{10}$$

$$x = 2 + \sqrt{10} \text{ only}$$

55. Solve the radical equation.

Solution

$$x^2 - 5x - 6 = 0$$

$$x = 6; \text{ reject } -1$$

56. Solve the area problem.

Solution

$$2x^2 + x - 93 = 0$$

$$x = \frac{-1 + \sqrt{745}}{4}$$

57. Prove the claim.

Solution
 Among three consecutive integers, one is even;
 therefore their product is even.

58. Make x the subject.

Solution

$$Q^{1/4} - b = ax^{-1/3}$$

$$x = \left(\frac{a}{Q^{1/4} - b} \right)^3$$

59. Make x the subject.

Solution

$$bx + c = \left(\frac{a}{R} \right)^{2/3}$$

$$x = \frac{(a/R)^{2/3} - c}{b}$$

60. Solve using $t = \sqrt{x}$.

Solution

$$2t^2 - 5t + 2 = 0$$

$$x = 4 \text{ or } \frac{1}{4}$$