Conditional and independent events

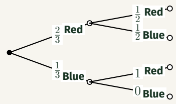
Decide whether events are independent or conditional, and calculate probabilities using trees, a Venn diagram, and a two-way table.

Name ______________________________ **Date** ______________________

Independent vs conditional

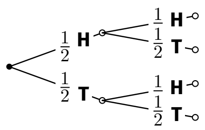
Events are INDEPENDENT if one does not affect the other's probability: $P(B \mid A) = P(B)$. Events are CONDITIONAL (dependent) if one changes the other's probability: $P(B \mid A) \neq P(B)$, read "the probability of B given A". Without replacement usually creates dependence; with replacement usually keeps events independent.

Worked example A bag has 2 red, 1 blue counter, drawn WITHOUT replacement. Find $P(\text{2nd is Red} \mid \text{1st was Red})$.



After removing a red counter, 1 red and 1 blue remain (2 total). $P(\text{2nd Red} \mid \text{1st Red}) = \frac{1}{2}$.

1. Two fair coins are tossed. Find $P(\text{Heads then Heads})$.



Solution
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

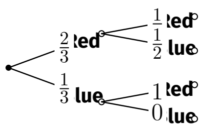
2. A die is rolled twice. Are the two rolls independent? Why?

Solution
Yes – the 1st roll doesn't change the 2nd die's faces or chances.

3. What does $P(B \mid A)$ mean, in words?

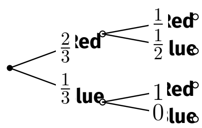
Solution
The probability of B, given that A has already happened.

4. Bag: 2 red, 1 blue (no replacement). Find $P(\text{2nd Red} \mid \text{1st Red})$.



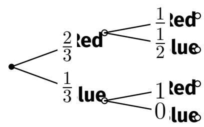
Solution
$P = \frac{1}{2}$ (1 red, 1 blue left).

5. Using the same tree, find $P(\text{2nd Red} \mid \text{1st Blue})$.



Solution
$P = 1$ (both left are red).

6. Find $P(\text{Red then Red})$ by multiplying along the branches.



Solution
$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

7. $P(B) = 0.5$ and $P(B \mid A) = 0.5$. Are A and B independent?

Solution
Yes – $P(B \mid A) = P(B)$.

8. $P(B) = 0.5$ but $P(B \mid A) = 0.8$. Are A and B independent? Explain.

Solution
No – $P(B \mid A) \neq P(B)$, so A changes B's probability.

9. Why does comparing $P(B)$ to $P(B \mid A)$ test for independence?

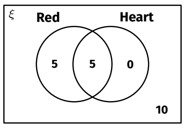
Solution
If they're equal, A has no effect on B (independent); if different, A changes B (dependent).

Conditional and independent events (continued)

Decide whether events are independent or conditional, and calculate probabilities using trees, a Venn diagram, and a two-way table.

Name ______________________ **Date** ______________________

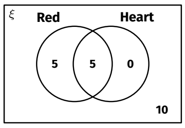
10. Of 20 cards, state $n(\text{Red})$ and $n(\text{Heart})$.



Solution

$n(\text{Red}) = 10$. $n(\text{Heart}) = 5$.

11. Find $P(\text{Heart})$ and $P(\text{Heart} \mid \text{Red})$. Independent?



Solution

$P(\text{Heart}) = \frac{1}{4}$,
$P(\text{Heart} \mid \text{Red}) = \frac{1}{2}$: dependent.

12. Why does knowing a card is Red change the probability it's a Heart?

Solution

All Hearts are Red, so restricting to Red removes the (non-Heart) black cards, raising the Heart share.

13. State $n(\text{Plays sport})$ and $n(\text{Tidy room})$ totals.

Sport/Tidy	Tidy	Not tidy	Total
Sport	15	25	40
No sport	10	50	60
Total	25	75	100

Solution

$n(\text{Sport}) = 40$. $n(\text{Tidy}) = 25$.

14. Find $P(\text{Tidy} \mid \text{Sport})$ vs $P(\text{Tidy})$ overall. Independent?

Sport/Tidy	Tidy	Not tidy	Total
Sport	15	25	40
No sport	10	50	60
Total	25	75	100

Solution

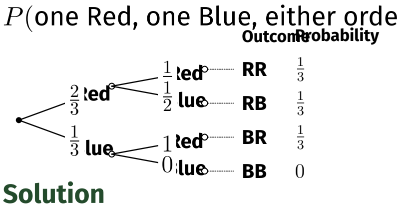
$P(\text{Tidy} \mid \text{Sport}) = 0.375$.
$P(\text{Tidy}) = 0.25$. Dependent.

15. Why is a two-way table useful for spotting conditional probability?

Solution

Dividing a joint cell by its row/column total directly gives a conditional probability.

16. Using the bag tree, find $P(\text{one Red, one Blue, either order})$.



Solution

$P(\text{RB}) + P(\text{BR}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.

17. "I already rolled a 6, so I'm less likely to roll one next time." Explain the error.

Solution

Gambler's fallacy: rolls are independent, so it's still $\frac{1}{6}$ every time.

18. Describe one example of independent events and one of dependent (conditional) events.

Solution

Independent: two separate coin tosses. Dependent: drawing two cards without replacement.