Key focus

This objective introduces the volume formulas for pyramids ($\frac{1}{3}\times$ base area $\times$ height), cones ($\frac{1}{3}\pi r^2 h$), and spheres ($\frac{4}{3}\pi r^3$), including the hemisphere as half a sphere. Students substitute correctly, work with π in exact and decimal form, and rearrange each formula to solve reverse problems (given the volume, find a missing radius, height, or base length).

Suggested teaching sequence

1. Give students all three formulas rather than expecting recall; the assessable skill is correct substitution and working, not memorisation.
2. Connect each formula to a prism or cylinder of the same base and height: a pyramid or cone is exactly one third of the matching prism or cylinder, and a hemisphere is exactly half a sphere.
3. Model substitution carefully, keeping π symbolic (e.g. leave as 48π) until a question explicitly asks for a decimal.
4. Introduce the slant-height cone: students must first use Pythagoras' theorem to find the perpendicular height from the radius and slant height before applying the volume formula.
5. Introduce reverse problems: given the volume and some dimensions, rearrange the formula to isolate the unknown (radius, height, or base side) before substituting. Model isolating the unknown as a separate step.
6. Introduce scale-factor reasoning: if every length of a solid is multiplied by k, the volume is multiplied by k^3 (not k^2, which students often confuse with the area rule).
7. Compare solids that share a dimension (e.g. a cone and a cylinder with the same radius and height) to reinforce the $\frac{1}{3}$ relationship, and solids with equal volumes to build estimation and equation-forming skills.
8. Apply to composite solids, such as a cone joined to a hemisphere, by finding each part's volume separately and adding.

Core examples to model

- Direct substitution: A cone has radius 4 cm and height 9 cm. $V = \frac{1}{3}\pi(4)^2(9) = 48\pi$ cm^3.
- Sphere in exact form: A sphere has radius 6 cm. $V = \frac{4}{3}\pi(6)^3 = 288\pi$ cm^3.
- Slant height first: A cone has radius 3 cm and slant height 5 cm. Perpendicular height $= \sqrt{5^2 - 3^2} = 4$ cm, so $V = \frac{1}{3}\pi(3)^2(4) = 12\pi$ cm^3.
- Reverse problem: A cone has volume 100π cm^3 and radius 5 cm. $100\pi = \frac{1}{3}\pi(5)^2 h$, so $h = 12$ cm.

- Reverse problem with a rearrangement: A square pyramid has volume 288 cm^3 and height 6 cm. $288 = \frac{1}{3}x^2(6)$, so $x^2 = 144$ and $x = 12$ cm.
- Composite solid: A toy joins a hemisphere (radius 4 cm) to a cone (radius 4 cm, height 9 cm). Hemisphere volume $= \frac{2}{3}\pi(4)^3 = \frac{128}{3}\pi$; cone volume $= \frac{1}{3}\pi(4)^2(9) = 48\pi$; total $= \frac{128}{3}\pi + 48\pi = \frac{272}{3}\pi$ cm^3.
- Scale factor: A sphere has radius 3 cm. If every length doubles, the volume is multiplied by $2^3 = 8$, not $2^2 = 4$.

Watch for

- Forgetting the $\frac{1}{3}$ (or $\frac{2}{3}$ for a hemisphere): students often apply the cylinder or sphere formula directly to a cone, pyramid, or hemisphere and overstate the volume.
- Using slant height instead of perpendicular height: the volume formula needs the perpendicular height; a given slant height must be converted first with Pythagoras' theorem.
- Confusing the scale factor: doubling a length multiplies volume by 8 (2^3), not 4 (2^2, the area rule) or 2.
- Early rounding: if π is rounded too soon, or an intermediate cube root or square root is rounded before the final step, the final answer can drift; keep exact values until the last line unless told to round earlier.
- Mismatching base shape and formula: a rectangular pyramid uses base area $=$ width $\times$ depth, not a squared side length; only a square-based pyramid uses x^2.
- Composite solids: forgetting that a joined solid's total volume is the sum of its parts, or accidentally double-counting the shared circular face.

Checks for understanding

- Can students recall and correctly substitute into the cone, pyramid, and sphere (and hemisphere) volume formulas?
- Can they find a cone's perpendicular height from its radius and slant height before finding the volume?
- Can they rearrange a volume formula to solve for a missing radius, height, or base side length?
- Can they explain why a pyramid or cone is one third of a matching prism or cylinder, and a hemisphere is half a sphere?
- Can they apply the k^3 scale-factor rule when every length of a solid is multiplied by a constant?
- Can they keep an answer in terms of π when asked, and round correctly to a stated number of decimal places otherwise?
- Can they find the total volume of a composite solid made of two joined shapes?