Key focus

This objective builds from reading and shading two-set Venn diagrams through to three-set diagrams, set notation cardinalities ($n(A)$, $n(A \cup B)$, $n(A \cap B)$, $n(A')$), and combining Venn diagrams with basic algebra. Not covered in the Beta textbook; resources are drawn entirely from Dr Austin Maths practice grids. Cardinality questions should be practised alongside the shading work throughout, not treated as a separate topic taught afterwards.

Suggested teaching sequence

1. Introduce a two-set Venn diagram with values already filled in. Model reading each region's value directly (in one set only, in both, in neither).
2. Ask what a shaded region represents in words, before introducing any set notation symbols.
3. Introduce $n(A)$ notation: the *total* in a set, including any overlap – not just the "only" region.
4. Introduce $n(A \cap B)$ (the overlap) and $n(A \cup B)$ (everything in at least one set), and $n(A')$ (the complement – everything not in A).
5. Practise working backwards: given a stated total and some known regions, find a missing region.
6. Introduce the inclusion-exclusion formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, and have students verify it against a diagram they've already read directly.
7. Extend to three-set Venn diagrams: model reading regions that belong to two sets (e.g. A and B but not C) and the single region belonging to all three.
8. Combine Venn diagrams with algebra: represent an unknown region with a letter, use the stated total to form an equation, and solve for it.

Core examples to model

- Reading directly: a two-set diagram with 7 in Football only, 4 in both, 6 in Netball only, and 3 in neither – $n(\text{Football}) = 7 + 4 = 11$, not just 7.
- Working backwards: 25 students, $n(\text{Maths}) = 15$, $n(\text{Science}) = 12$, $n(\text{both}) = 6$ – Maths only $= 9$, Science only $= 6$, neither $= 25 - 21 = 4$.
- Union formula check: $n(A) = 10$, $n(B) = 8$, $n(A \cap B) = 3$ gives $n(A \cup B) = 10 + 8 - 3 = 15$.
- Algebra with Venn: an unknown region x, with the rest of the regions and the total given, solved as a simple linear equation.

Watch for

- Confusing "$n(A)$" with just the "A only" region – $n(A)$ always includes any overlap with other sets.
- Adding the wrong two regions together (e.g. adding A only to B only to try to get $n(A)$) instead of A only plus the overlap.

- Forgetting that $n(A')$ (the complement) includes everyone *outside* A, including the "neither" region, not just B only.
- Double-counting the overlap when adding regions for $n(A \cup B)$ instead of using A only $+$ B only $+$ overlap (or the union formula).
- In a three-set diagram, missing that the very centre region belongs to all three sets at once, not just two.

Checks for understanding

- Can students read a region's value directly off a two- or three-set Venn diagram?
- Can they describe a shaded region in words before using notation?
- Can they calculate $n(A)$, $n(A \cup B)$, $n(A \cap B)$, and $n(A')$ correctly?
- Can they work backwards from a stated total to find one missing region?
- Can they apply and verify the inclusion-exclusion formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$?
- Can they read cardinalities from a three-set diagram, including two- and three-way overlaps?
- Can they represent an unknown region algebraically and solve for it using a stated total?