Teacher guidance

Using algebra to find the point of intersection of two lines

Key focus

When two straight lines meet, they intersect at one point. That point is the only (x, y) coordinate that satisfies both line equations simultaneously. To find it, we equate the two equations and solve.

How to find the intersection

1. If both lines are in the form $y = mx + c$, equate them: $y_1 = y_2$.
2. Rearrange to collect all variable terms on one side.
3. Solve for x.
4. Substitute the x-value into either equation to find y.

Key points to remember

- Two different lines meet at exactly one point
- At the intersection, both equations give the same y
- Always check your answer by substituting into both equations
- Lines can be given in different forms: rearrange if needed
- The intersection point has coordinates (x, y) where x is from solving, y is from substituting

Example 1: both lines in $y = mx + c$ form

$y = x + 2$ and $y = 5 - x$. Equate: $x + 2 = 5 - x$. Add x to both sides: $2x + 2 = 5$. Subtract 2: $2x = 3$, so $x = 1.5$. Substitute back: $y = 1.5 + 2 = 3.5$. Intersection: $(1.5, 3.5)$. Check in second equation: $y = 5 - 1.5 = 3.5$?

Example 2: one line rearranged

$y = 2x + 1$ and $3x + y = 11$. Rearrange the second: $y = 11 - 3x$. Equate: $2x + 1 = 11 - 3x$. Add $3x$: $5x + 1 = 11$. Subtract 1: $5x = 10$, so $x = 2$. Substitute: $y = 2(2) + 1 = 5$. Intersection: $(2, 5)$.

Example 3: fractional solutions

$y = 3x - 1$ and $y = -\frac{1}{2}x + 4$. Equate: $3x - 1 = -\frac{1}{2}x + 4$. Add $\frac{1}{2}x$: $3.5x - 1 = 4$ or $\frac{7}{2}x = 5$. So $x = \frac{10}{7}$. Substitute: $y = 3 \cdot \frac{10}{7} - 1 = \frac{30}{7} - \frac{7}{7} = \frac{23}{7}$.

Example 4: checking the answer

Always verify! For $y = 2x - 3$ and $y = -x + 9$ with intersection $(4, 5)$: First equation: $y = 2(4) - 3 = 5$? Second equation: $y = -(4) + 9 = 5$? Both give the same y, so the answer is

correct.

Try these

1. Find where $y = x + 3$ and $y = 9 - x$ intersect.
2. Find where $y = 2x - 1$ and $x + y = 8$ intersect.
3. Find where $2x + y = 10$ and $y = x - 1$ intersect.
4. Find where $y = \frac{1}{2}x + 2$ and $y = 2x - 1$ intersect.

Watch for

Common errors: (1) forgetting to substitute back to find y—finding x is only halfway! (2) Not checking the answer—substitution into both equations catches most mistakes. (3) Sign errors when rearranging—be careful with subtraction.