Key focus

This is the extension objective paired directly with the introductory "finding a side" objective: instead of a known angle and side giving a missing side, students are now given two known sides and must find the missing angle using the inverse trigonometric functions ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$). Students first label the hypotenuse, opposite, and adjacent sides relative to the unknown angle, then decide which pair of known sides they have (opposite/hypotenuse, adjacent/hypotenuse, or opposite/adjacent) to choose sin, cos, or tan, before applying the inverse function to solve for the angle.

Suggested teaching sequence

1. Revisit labelling the hypotenuse, opposite, and adjacent sides relative to a marked (here, unknown) angle, in different orientations.
2. Revisit SOH CAH TOA, this time emphasising that with two known sides the ratio itself can be calculated as a decimal even before the angle is known.
3. Model finding a missing angle: write the ratio, substitute the two known sides, then apply the inverse function – for example $\sin(x) = 7/14 = 0.5$, so $x = \sin^{-1}(0.5) = 30^\circ$.
4. Practise with a calculator in degree mode, using the $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ keys (often accessed as shift/2nd-function of sin, cos, tan). Emphasise checking degree mode before starting.
5. Introduce a stated rounding accuracy (1 decimal place unless told otherwise) and model writing the unrounded ratio before rounding the final angle only.
6. Extend to real-world contexts: ladders, ramps, kite strings, and guy-wires where two side lengths are given and the angle they make must be found.
7. Introduce angle of elevation (measured up from the horizontal at the observer) and angle of depression (measured down from the horizontal at an elevated observer) as specific real-world framings of the same angle-finding skill.
8. Introduce the exact trigonometric ratios for 0°, 30°, 45°, 60°, and 90° so students can recognise, for example, that $\tan(x) = 1$ gives $x = 45^\circ$ or $\sin(x) = \frac{\sqrt{3}}{2}$ gives $x = 60^\circ$ without a calculator.
9. Practise multi-step problems that chain a find-a-side step (from the paired objective) into a find-an-angle step, making sure students clearly label and carry forward the value found in the first stage.

Core examples to model

- Finding the angle from opposite and hypotenuse: opposite 8 cm, hypotenuse 19 cm. $x = \sin^{-1}(8/19) \approx 24.9^\circ$ (1 d.p.).
- Finding the angle from adjacent and hypotenuse: adjacent 16 cm, hypotenuse 19 cm. $x = \cos^{-1}(16/19) \approx 32.6^\circ$ (1 d.p.).
- Finding the angle from opposite and adjacent: opposite 6 cm, adjacent 14 cm.

$x = \tan^{-1}(6/14) \approx 23.2°$ (1 d.p.).

- Angle of elevation: a drone is 22 m above the ground, directly above a point 38 m, measured horizontally, from the observer. $x = \tan^{-1}(22/38) \approx 30.07°$ (2 d.p.).
- Angle of depression: from the top of a 26 m cliff, a boat is 60 m, measured horizontally, from the base. $x = \tan^{-1}(26/60) \approx 23.43°$ (2 d.p.).
- Exact ratio: $\tan(x) = 1 \Rightarrow x = 45°$; $\sin(x) = \frac{\sqrt{3}}{2} \Rightarrow x = 60°$; $\cos(x) = \frac{\sqrt{3}}{2} \Rightarrow x = 30°$.
- Multi-step problem: a ladder 9.4 m long leans at $63°$, reaching a height of $9.4\sin(63°) \approx 8.38$ m. A second observer 3.1 m from the wall then sights the same point: angle of elevation $= \tan^{-1}(8.38/3.1) \approx 69.69°$.

Watch for

- Choosing the wrong ratio: the most common error is picking sin, cos, or tan without first correctly identifying which two given sides (relative to the unknown angle) are actually involved.
- Forgetting the inverse step: writing $\sin(x) = 0.5$ and stopping, instead of applying $\sin^{-1}$ to both sides to isolate x.
- Confusing opposite and adjacent when the triangle is rotated or mirrored: both are defined relative to the unknown angle, not to the page orientation.
- Calculator in the wrong mode: an answer that looks like radians (a small decimal instead of a sensible number of degrees) usually means the calculator is in radian mode, not degree mode.
- Entering an impossible ratio: $\sin^{-1}$ and $\cos^{-1}$ are undefined for any value outside -1 to 1, since sine and cosine of an angle can never exceed 1 in magnitude – a calculator error usually signals a mislabelled side, not a genuine "no solution."
- Rounding too early: rounding the ratio before applying the inverse function can shift the last decimal place of the angle answer.
- For elevation/depression problems, measuring the angle from the wrong reference line: elevation is measured from the horizontal at the lower (ground-level) observer; depression is measured from the horizontal at the higher (elevated) observer – not at the object being sighted.

Checks for understanding

- Can students correctly label hypotenuse, opposite, and adjacent relative to an unknown marked angle in any triangle orientation?
- Can they choose the correct ratio (sin, cos, or tan) from two given sides before calculating?
- Can they set up the ratio and correctly apply the inverse trig function to solve for the angle?
- Can they round a final angle to a stated accuracy without rounding the ratio itself first?
- Can they recall the exact angle for a stated exact ratio (e.g. $\tan(x) = 1$) without a calculator?
- Can they apply the skill to a real-world elevation or depression scenario, correctly identifying where the unknown angle sits?
- Can they work through a two-step problem where a missing side must be found before it can be used to find a missing angle?