Multi-step Pythagoras theory

A structured approach to problems requiring two or more steps

Key focus

A multi-step Pythagoras question uses Pythagoras' theorem to find an intermediate length first, then uses that result in a second step such as area, perimeter, a comparison, or another right-angled triangle.

The typical pattern

1. Find the hidden right-angled triangle that needs Pythagoras' theorem.
2. Use Pythagoras to find the missing side, height, or diagonal.
3. Use that new value in the second step.
4. Finish with the real target and the correct units.

Second-step operations

Students might need to:

- Calculate the area of a triangle or other shape
- Calculate the perimeter of a shape
- Compare a diagonal route with walking around two sides
- Use a first diagonal inside a second right triangle
- Work with exact versus rounded answers

Example 1: height then area

An isosceles triangle has equal sides 13 cm and base 10 cm.
Split the base in half, so each half is 5 cm.
Using Pythagoras: $h^2 = 13^2 - 5^2 = 169 - 25 = 144$, so $h = 12$ cm.
Now use the height to find area: Area $= \frac{1}{2} \times 10 \times 12 = 60$ cm^2.

Example 2: diagonal then perimeter

A rectangle is 6 cm by 8 cm.
Using Pythagoras: $d^2 = 6^2 + 8^2 = 36 + 64 = 100$, so $d = 10$ cm.
Perimeter $= 2(6 + 8) = 28$ cm. (Note: the diagonal was found but wasn't the final answer.)

Watch for

Students often stop once they have found the diagonal or height. Emphasise that if the question asks for area, perimeter, or a comparison, they must use the intermediate value to complete the second step. Encourage them to write the intermediate answer clearly so they don't lose track of which value goes where.

Curriculum content

Foundation: Diagram-led problems with whole-number values and cleanly-integer answers. Word problems involving rectangles, squares, and isosceles triangles. Questions are mostly geometric; word/text problems are a minority.

Proficient: Decimal-scaled triangles and rectangles; rounding to 1 d.p. when specified. More diverse real-world contexts (ladders, boats, screens, pitches). Mix of diagram and word problems.

Excellence: Complex calculations with surds and exact answers. Problems requiring three or more steps. Coordinate geometry, hexagons, and reasoning questions that ask students to explain why a particular step is needed.

Example 3: compare two routes

A rectangle is 8 m by 15 m.
Diagonal: $d^2 = 8^2 + 15^2 = 64 + 225 = 289$, so $d = 17$ m.

Walking two sides: $8 + 15 = 23$ m.

The diagonal is $23 - 17 = 6$ m shorter.

Example 4: square diagonal then side work

A square has diagonal 18 cm. If the side is s, then $s^2 + s^2 = 18^2$.
$2s^2 = 324$, so $s^2 = 162$, so $s = 9\sqrt{2}$ cm.

Now use that side in perimeter or area: Perimeter $= 4 \times 9\sqrt{2} = 36\sqrt{2}$ cm.