Key focus

This is the introductory Pythagoras' theorem objective. It is deliberately scoped to just one skill – finding a missing hypotenuse (the long side) of a right-angled triangle – so students master the core relationship $a^2 + b^2 = c^2$ before the fuller objective (finding either a missing hypotenuse or a missing shorter side) asks them to decide which direction to work in. Students identify the hypotenuse as the longest side, opposite the right angle, then square the two known shorter sides, add, and square root. Not every student needs to work fully algebraically straight away; the area model – the square on the hypotenuse equals the sum of the squares on the two shorter sides – supports understanding before moving to direct algebraic use of the formula.

Suggested teaching sequence

1. Introduce the area model: draw a right-angled triangle with a square on each side. Show that the area of the square on the longest side equals the sum of the areas of the squares on the two shorter sides.
2. Identify the hypotenuse: always the longest side, always opposite the right angle. Practise labelling it in triangles drawn in different orientations (not just with the right angle at the bottom-left).
3. Introduce the formula $a^2 + b^2 = c^2$, where c is the hypotenuse. Model finding a missing hypotenuse: square both known sides, add, then square root.
4. Practise with whole-number Pythagorean triples first (3-4-5, 5-12-13, 8-15-17, 7-24-25, 9-40-41 and their multiples) so students can check their answers land on a whole number.
5. Extend to decimal side lengths, then to answers that must be rounded to a stated number of decimal places.
6. Introduce non-perfect-square results: show that an answer such as $\sqrt{34}$ or $\sqrt{65}$ is left as a simplified surd unless the question asks for a rounded decimal. Practise simplifying, e.g. $\sqrt{50} = 5\sqrt{2}$.
7. Connect to the area form: given the two shorter-side square areas, add them to find the hypotenuse's square area, then square root only at the end to recover a length.
8. Apply to real contexts where the unknown is explicitly the longest side or diagonal: a diagonal brace across a rectangular gate, the diagonal of a rectangle, or the straight-line distance between two points reached by travelling along two perpendicular legs.

Core examples to model

- Finding the hypotenuse: shorter sides 6 cm and 8 cm. $c = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ cm.
- Using square areas directly: the short-side squares have areas 64 cm^2 and 225 cm^2; the hypotenuse's square area is $64 + 225 = 289$, so the hypotenuse is 17 cm.
- Rounding to a stated accuracy: legs 5.3 m and 7.9 m give $c = \sqrt{28.09 + 62.41} = \sqrt{90.5} \approx 9.5$ m (1 d.p.).

- Non-perfect-square (surd) answer: legs 3 and 5. The hypotenuse is $\sqrt{3^2+5^2}=\sqrt{34}$; left as $\sqrt{34}$ unless a rounded value is requested.
- Simplifying a surd: legs 2 and 4 give $c=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$, since $20=4\times 5$ and 4 is a perfect square.
- Diagonal of a rectangle: a 3 m by 9 m rectangle has diagonal $\sqrt{3^2+9^2}=\sqrt{90}=3\sqrt{10}$ m.

Watch for

- Forgetting to square root: after adding the two squares, students often report the squared value (for example 100) instead of the side length (10).
- Adding lengths instead of squares: a common error is $6+8=14$ instead of $6^2+8^2=100$, then $\sqrt{100}=10$.
- Misidentifying the hypotenuse when the triangle is rotated or mirrored: the hypotenuse is always opposite the right angle, regardless of which side of the diagram it is drawn on.
- Leaving a decimal answer where a surd was expected, or vice versa: check whether the question specifies rounding (e.g. "to 1 d.p.") before deciding the answer format.
- Incompletely simplifying a surd: $\sqrt{20}$ left as-is instead of simplified to $2\sqrt{5}$ – check for a perfect-square factor before finishing.
- Confusing the area-of-the-square language ("the square on the hypotenuse has area 289") with the side length itself (the side length is $\sqrt{289}=17$, not 289).

Checks for understanding

- Can students correctly identify the hypotenuse in triangles drawn in any orientation?
- Can they find a missing hypotenuse by adding the squares of the two shorter sides and square rooting?
- Can they move between the area form (square areas) and the length form (side lengths) of the theorem?
- Can they round an answer to a stated number of decimal places when asked, and leave an exact surd when not?
- Can they simplify a surd such as $\sqrt{50}$ to $5\sqrt{2}$ by finding a perfect-square factor?
- Can they set up and solve a real-world hypotenuse problem (a diagonal brace, a rectangle's diagonal, a straight-line distance) from a written description?