Overview

This learning objective focuses on students' understanding of interior (internal) and exterior (external) angles of polygons. Students progress from triangle angle relationships through to irregular polygons and regular polygon calculations using the formula for interior angles.

Key Concepts

Angles in a Triangle

The sum of interior angles in any triangle is always $180°$. This is a fundamental concept that students must understand deeply. Help students visualize this by:

- Drawing triangles of different types (acute, right, obtuse)
- Showing that if you cut out the three angles and place them adjacent on a straight line, they form exactly $180°$

Angles in Quadrilaterals

The sum of interior angles in any quadrilateral is $360°$. This can be understood as two triangles ($2 \times 180° = 360°$), or as a straight angle doubled.

Polygon Angle Sum Formula

For any polygon with n sides:

$$\text{Sum of interior angles} = (n - 2) \times 180°$$

For a regular polygon (all angles equal):

$$\text{Each interior angle} = \frac{(n - 2) \times 180°}{n}$$

Interior and Exterior Angle Relationship

At any vertex of a polygon:

$$\text{Interior angle} + \text{Exterior angle} = 180°$$

An exterior angle is formed when one side of the polygon is extended past a vertex. The exterior angle and its adjacent interior angle form a linear pair (supplementary angles on a straight line).

Common Misconceptions

1. **Confusing interior and exterior angles:** Students may think the exterior angle is "outside" rather than understanding it as the supplement to the interior angle. Emphasize that the exterior angle shares a side with the interior angle at each vertex.
2. **Applying wrong polygon angle sum:** Students might use $n \times 180°$ instead of $(n-2) \times 180°$. Remind them that a pentagon has 5 angles summing to 540° (not 900°).
3. **Assuming all polygons are regular:** Emphasize that a polygon is regular ONLY when all sides and angles are equal. An irregular polygon with the same number of sides will have the same angle sum, but the individual angles vary.
4. **Not recognizing isosceles triangle properties:** Remind students that in an isosceles triangle, the two base angles are equal, which helps solve for the unknown angle.

Teaching Sequence

1. **Week 1: Triangle Angles**
   - Start with discovery activity: cut out and rearrange triangle corners
   - Practice finding missing angles in simple right triangles
   - Extend to general triangles with two known angles
2. **Week 2: Quadrilaterals and the $360°$ Rule**
   - Build on triangle understanding by showing a quadrilateral is two triangles
   - Practice with rectangles (all $90°$) before irregular quadrilaterals
   - Solve problems with multiple unknown angles (e.g., $x + 10°$, $x + 20°$)
3. **Week 3: Exterior Angles**
   - Show the relationship: interior $+$ exterior $= 180°$
   - Practice identifying exterior angles in diagrams
   - Solve problems finding either interior or exterior angles
4. **Week 4: Regular Polygons**
   - Introduce the polygon angle sum formula
   - Calculate interior angles of regular pentagons, hexagons, octagons
   - Reverse problems: given an interior angle, find the number of sides

Worked Examples

Example 1: A triangle has angles of $55°$, $65°$, and x. Find x.

$$55° + 65° + x = 180°$$

$$x = 180° - 55° - 65° = 60°$$

Example 2: An isosceles triangle has equal base angles of x and an apex angle of $40°$. Find x.

$$x + x + 40° = 180°$$

$$2x = 140°, \quad x = 70°$$

Example 3: What is each interior angle of a regular pentagon?

$$\text{Sum} = (5 - 2) \times 180° = 3 \times 180° = 540°$$

$$\text{Each angle} = \frac{540°}{5} = 108°$$

Example 4: If an interior angle of a regular polygon is $120°$, how many sides does it have?

$$\frac{(n-2) \times 180°}{n} = 120°$$

$$(n-2) \times 180° = 120n$$

$$180n - 360° = 120n$$

$$60n = 360°, \quad n = 6 \text{ (hexagon)}$$

Assessment Tips

Look for students who:

- Can correctly identify angles in diagrams
- Apply the angle sum formula consistently
- Can work backwards from angles to find the number of polygon sides
- Understand why exterior angles are always $180°$ minus the interior angle
- Can solve problems with algebraic expressions for angles