Key focus

This objective develops students' ability to translate real-world situations and word problems into algebraic equations, then solve them and interpret the solutions. The key principle is identifying the unknown quantity, defining a variable to represent it, writing an equation based on the problem description, solving the equation using inverse operations, and verifying that the solution makes sense within the original context. Students learn that equations are tools for solving practical problems, and that the same equation-solving techniques apply whether the equation comes from a symbolic prompt or from a real-world scenario. Emphasis should be placed on systematic translation of words to symbols, maintaining balance during solving, and always checking that answers are reasonable.

Suggested teaching sequence

1. Start with simple contexts and linear relationships: "Ben is x years old. His father is three times Ben's age minus 5. If the father is 40, find Ben's age." Translate step-by-step: father's age is $3x - 5$, and this equals 40.
2. Introduce geometric contexts (perimeter, area): "A rectangle has length $2x + 1$ and width x. If the perimeter is 32 m, find the width." Emphasise the formula $P = 2(l + w)$ and substitution.
3. Show comparison contexts (two companies, speeds, ages): "Company A charges \$10 + \$15/hr, Company B charges \$5 + \$16/hr. When do they cost the same?" Set up $10 + 15h = 5 + 16h$ and solve.
4. Introduce multi-step translations: "The sum of three consecutive integers is 45. Find the integers." Use x, $x + 1$, $x + 2$ and solve $x + (x + 1) + (x + 2) = 45$.
5. Show verification by substitution: always substitute the solution back into the original equation and the original word problem. Ask: "Does the answer make sense in the context? Are there any constraints (e.g., age must be positive)?"
6. Introduce coefficient interpretation: "A book costs \$8 more than a pen. If 2 books and 3 pens cost \$46, find the pen price." Emphasise the algebraic structure: let p = pen cost, then book cost is $p + 8$.
7. Show more complex relationships (quadratic, rational): context may require equations beyond linear (e.g., area of a right triangle, time for pipes to fill a tank).
8. Emphasise common errors: misidentifying the variable, forgetting to set up the equation correctly, solving correctly but misinterpreting the answer, or failing to verify in context.

Core examples to model

- Simple age problem: "Ben is x years old. His father is $3x - 5$ and equals 40." Write $3x - 5 = 40$, solve to get $x = 15$, verify: $3(15) - 5 = 40$ ✓.
- Perimeter problem: "Rectangle: length $2x + 1$, width x, perimeter 32." Write $2(2x + 1 + x) = 32$, simplify to $6x + 2 = 32$, solve to get $x = 5$ m, verify: $2(11 + 5) = 32$ ✓.

- Comparison: ”Company A: \$10 + \$15/h; Company B: \$5 + \$16/h. Equal cost?” Write $10 + 15h = 5 + 16h$, solve to get $h = 5$ hours, verify in both formulas: $10 + 75 = 85 = 5 + 80$ ✓.
- Consecutive integers: ”Sum of three consecutive integers is 45.” Write $x + (x + 1) + (x + 2) = 45$, solve to get $x = 14$, so integers are 14, 15, 16, verify: $14 + 15 + 16 = 45$ ✓.
- Money problem: ”Book is \$8 more than pen. 2 books + 3 pens = \$46. Find pen price.” Let p = pen, then book = $p + 8$. Write $2(p + 8) + 3p = 46$, solve to get $p = 6$ dollars, verify: $2(14) + 3(6) = 28 + 18 = 46$ ✓.

Watch for

- Forgetting what the variable represents: always define the variable clearly at the start. If x is ”the width,” keep that meaning throughout; don’t switch to ”the length” partway through.
- Misreading the problem: ”three times Ben’s age minus 5” means $3x - 5$, not $3(x - 5)$. Parse carefully and consider asking students to underline the key relationships.
- Setting up the wrong equation: ”The sum of a number and its double is 36” means $n + 2n = 36$, not $n + 2 = 36$ or $2n = 36$.
- Solving correctly but forgetting context: solving $3x - 5 = 40$ to get $x = 15$ is correct, but students must then say ”Ben is 15 years old,” not just ”\$ = 15” or ”answer is 15.”
- Failing to verify: after solving, always substitute back into the ORIGINAL equation and check that the answer makes sense in the problem context.
- Negative or non-integer solutions that contradict context: if a problem asks for the number of students and the solution is -5 or 3.5, the problem may have no valid solution, or the context may need reinterpretation.
- Forgetting to define multiple variables: if a problem mentions ”books” and ”pens,” clearly define b = book cost and p = pen cost before writing equations.
- Rushed algebraic steps: encourage showing all steps, especially the substitution and simplification, to catch errors early.

Checks for understanding

- Can students identify the unknown quantity and choose an appropriate variable?
- Can they translate a word problem into an algebraic equation?
- Can they solve the resulting equation using inverse operations and like-term collection?
- Can they interpret the solution within the original problem context?
- Can they check that a solution is reasonable (e.g., age is positive, count is a whole number)?
- Can they verify a solution by substituting back into both the equation and the original problem statement?
- Can they handle equations arising from geometric contexts (perimeter, area)?
- Can they handle equations arising from comparison contexts (two prices, speeds, ages)?
- Can they identify and correct errors in a given translation or solution?