



Converting between improper fractions and mixed numbers

Key focus

An improper fraction (numerator $\geq$ denominator) and a mixed number (a whole number plus a proper fraction) are two ways of writing the same value that is one whole or more. Converting to a mixed number divides the numerator by the denominator: the quotient becomes the whole number and the remainder becomes the new numerator, over the same denominator. Converting to an improper fraction reverses this: multiply the whole number by the denominator, add the numerator, and keep the same denominator. This LO builds on Forming, Equivalent, and Simplifying Fractions (LO1-LO3) by extending fraction fluency beyond one whole.

Suggested teaching sequence

1. Show a mixed-bars diagram: a row of unit tapes, each split into the denominator's equal parts, with the numerator shaded left to right so the shading spills from one whole tape into the next.
2. Read the picture both ways: counting the fully-shaded tapes plus the leftover part gives the mixed number; counting every shaded part over the denominator gives the improper fraction.
3. Formalise the improper-to-mixed method: divide the numerator by the denominator; the quotient is the whole number, the remainder is the new numerator, and the denominator stays the same.
4. Formalise the reverse, mixed-to-improper method: multiply the whole number by the denominator, add the numerator, and keep the same denominator.
5. Practise both directions, checking against the bar model before working symbolically.
6. Address the boundary case where the numerator is an exact multiple of the denominator, so the mixed number has no fraction part (e.g. $\frac{8}{4} = 2$).
7. Compare a mixed number and an improper fraction by converting both to the same form first, then use this to order values or check a claimed conversion.

Core examples to model

- Bar model: $\frac{7}{4}$ shown as one fully-shaded tape plus $\frac{3}{4}$ of a second – $\frac{7}{4} = 1\frac{3}{4}$.
- Improper to mixed: $\frac{17}{5}$ – $17 \div 5 = 3$ remainder 2, so $\frac{17}{5} = 3\frac{2}{5}$.
- Mixed to improper: $5\frac{3}{7}$ – $5 \times 7 + 3 = 38$, so $5\frac{3}{7} = \frac{38}{7}$.
- Boundary case: $\frac{8}{4} = 2$ exactly, with no leftover fraction part.
- Comparing: $\frac{11}{3} = 3\frac{2}{3}$, which is greater than $3\frac{1}{2}$.

Watch for

- Changing the denominator when converting in either direction – it must stay the same throughout.
- Adding the numerator before multiplying by the denominator (or forgetting to add it at all) when building an improper fraction.
- Treating the remainder as the final answer, or the whole-number quotient as the fraction part.
- Comparing a mixed number and an improper fraction directly without first converting them to the same form.

Checks for understanding

- Can students convert an improper fraction to a mixed number by dividing, and explain what the remainder represents?
- Can they convert a mixed number to an improper fraction by multiplying the whole by the denominator and adding the numerator?
- Can they use a mixed-bars diagram to justify a conversion in either direction?
- Can they compare a mixed number and an improper fraction by converting both to a common form?
- Can they handle the boundary case where a conversion gives a whole number with no remainder?