Key focus

This LO distinguishes independent events (one event's outcome does not affect the probability of another, e.g. two separate dice rolls, where $P(A \cap B) = P(A) \times P(B)$) from conditional/dependent events (one event changes the probability of another, e.g. drawing a second card without replacement, written $P(B \mid A)$, "the probability of B given A"). The central test is comparing $P(B)$ with $P(B \mid A)$: if they are equal, A and B are independent; if different, they are dependent. Without-replacement trees, a Venn diagram's overlap, and two-way tables all give concrete ways to calculate and compare these probabilities.

Suggested teaching sequence

1. Introduce independent events with two fair coin tosses: the 1st toss cannot affect the 2nd, so $P(\text{HH}) = P(H) \times P(H)$.
2. Introduce dependent/conditional events with a without-replacement bag draw: removing a counter changes what's left, so the 2nd draw's probabilities depend on the 1st result. Build the tree explicitly, showing how the 2nd-stage branches change depending on which 1st-stage branch was taken.
3. Introduce $P(B \mid A)$ notation and read it aloud as "the probability of B, given A has happened." Contrast with the plain (unconditional) $P(B)$.
4. Establish the independence test: compute $P(B)$ and $P(B \mid A)$ separately and compare. Equal $\Rightarrow$ independent; different $\Rightarrow$ dependent.
5. Show the same test using a Venn diagram (e.g. Red cards and Hearts): $P(\text{Heart})$ overall vs. $P(\text{Heart} \mid \text{Red})$, reasoning from the overlap and the "outside" region.
6. Show the same test using a two-way table: a conditional probability is a joint cell divided by its row or column total (e.g. $P(\text{Tidy} \mid \text{Sport}) = \dfrac{\text{Sport-and-Tidy count}}{\text{Sport total}}$), which is easy to read directly off the table.
7. Combine branches in a without-replacement tree for combined-event questions: "one red, one blue, either order" adds two leaf probabilities; "at least one blue" uses the complement of "both red".
8. Address the gambler's fallacy directly: for genuinely independent events, an earlier outcome does not change a later probability, however tempting it feels.
9. Discuss real-world caution: a correlation in a survey (e.g. left-handed students who play guitar) is evidence of association, but should be checked against sample size and other explanations before concluding real dependence.

Core examples to model

- Independent: two fair coins, $P(\text{HH}) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$.
- Conditional (without replacement): a bag with 2 red, 1 blue counter – after removing a red,

$P(\text{2nd Red} \mid \text{1st Red}) = \frac{1}{2}$; after removing a blue, $P(\text{2nd Red} \mid \text{1st Blue}) = 1$.

- Comparing $P(B)$ and $P(B \mid A)$: $P(B) = 0.5$, $P(B \mid A) = 0.8$ – not equal, so A and B are dependent.
- Venn/overlap check: of 20 cards, $P(\text{Heart}) = \frac{5}{20} = \frac{1}{4}$ but $P(\text{Heart} \mid \text{Red}) = \frac{5}{10} = \frac{1}{2}$ – dependent, because every Heart is Red.
- Two-way table check: $P(\text{Tidy} \mid \text{Sport}) = \frac{15}{40} = 0.375$ vs. $P(\text{Tidy}) = \frac{25}{100} = 0.25$ – dependent.
- Combined events (without replacement): $P(\text{one Red, one Blue, either order}) = P(\text{RB}) + P(\text{BR}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.

Watch for

- Assuming any two events happening in sequence must be dependent – sequence alone does not imply dependence (e.g. two separate coin tosses).
- Forgetting that removing an item without replacement changes BOTH the total and, sometimes, the count of a specific outcome remaining.
- The gambler's fallacy: believing an independent event "owes" a different outcome after a run of the same result.
- Concluding dependence from a single small-sample survey observation without checking whether the association could be coincidence.
- Confusing $P(B \mid A)$ with $P(A \mid B)$ – these are generally different probabilities and swapping them is a common error.

Checks for understanding

- Can students explain the difference between independent and conditional (dependent) events, with an example of each?
- Can they build and read a without-replacement tree, where the 2nd-stage branches change based on the 1st-stage outcome?
- Can they test independence by comparing $P(B)$ with $P(B \mid A)$, using a tree, a Venn diagram, or a two-way table?
- Can they calculate a combined-event probability (e.g. "either order", "at least one") from a without-replacement tree?
- Can they critique a probability claim that confuses independence with dependence, including the gambler's fallacy?