Teacher guidance

Key focus

This objective asks students to form and fully simplify an algebraic expression for the area of a compound shape – an L-shape or a rectangle with a rectangular hole – whose side lengths are given as algebraic terms or expressions rather than numbers. It sits on top of two skills students already have: decomposing a compound shape into two rectangles (from the numeric composite-area objective) and expanding/collecting like terms (from earlier algebra work). The extension step asks students to set the simplified expression equal to a given area and solve the resulting linear equation for the unknown, then use that value in a follow-up calculation such as the perimeter.

Suggested teaching sequence

1. Revisit that the area of a rectangle is length $\times$ width, and practise this with one algebraic side, e.g. a rectangle 3 cm by x cm has area $3x$.
2. Show how an L-shape splits into two rectangles (as in the numeric composite-shapes objective), but now label the split with algebraic side lengths.
3. Model writing the two rectangle areas as a sum, e.g. $(3)(2x) + (7)(2)$, then simplifying to one expression, $6x + 14$.
4. Introduce binomial side lengths (e.g. $2x + 1$) so students must expand a bracket, e.g. $(3)(2x + 1) = 6x + 3$, before collecting like terms with the other rectangle's area.
5. Introduce the second shape family: a rectangle with a rectangular hole. Model that its area is the outer rectangle's area *minus* the hole's area, e.g. $(4x)(5) - (3)(2) = 20x - 6$.
6. Practise fully simplifying – expand every bracket, then collect like terms into one expression with no more simplifying possible.
7. Extend to "the area is a given number, find the unknown": set the simplified expression equal to the given area and solve the resulting linear equation.
8. Extend further to a follow-up step after solving for the unknown – substitute it back in to find the perimeter, or evaluate the simplified expression at a different given value.

Core examples to model

- Simplify only (L-shape): sides $2x$+2, 3, $2x$, 4, 2, 7 cm. Area $= (3)(2x) + (7)(2) = 6x + 14$.
- Simplify with a bracket to expand (L-shape): sides $2x$+5, 3, $2x$+1, 2, 4, 5 cm. Area $= (3)(2x + 1) + (5)(4) = 6x + 3 + 20 = 6x + 23$.
- Simplify with a hole to subtract: outer $4x$ cm by 5 cm, hole 3 cm by 2 cm. Area $= (4x)(5) - (3)(2) = 20x - 6$.
- Form, simplify, then solve (L-shape): sides x+3, 3, x, $2x$+1, 3, $2x$+4 cm. Area $= (3)(x) + (2x+4)(3) = 9x + 12$. If the area is 75 cm^2: $9x + 12 = 75 \Rightarrow x = 7$. Then the perimeter is $6x + 14 = 6(7) + 14 = 56$ cm.

Watch for

- Forgetting to distribute a coefficient across a bracket, e.g. writing $3(2x + 1) = 6x + 1$ instead of $6x + 3$.
- Adding the hole's area instead of subtracting it on the rectangle-with-hole shape.
- Stopping before collecting like terms – $6x + 3 + 20$ left unsimplified is not a final answer; it must become $6x + 23$.
- Multiplying the wrong pair of sides when splitting an L-shape into two rectangles – each rectangle needs one side from the outer edge and one from an adjacent labelled edge, not two arbitrary labels.
- Dropping the units (cm^2 for area, cm for a length or perimeter) in the final answer.
- After solving for the unknown, stopping without answering a follow-up part of the question (perimeter, or the area at another value).

Checks for understanding

- Can students identify the two rectangles that make up a pictured L-shape or rectangle-with-hole?
- Can they write each rectangle's area as a product of two labelled sides?
- Can they expand a bracket such as $3(2x + 1)$ correctly?
- Can they collect like terms to reach one fully simplified expression?
- Can they set a simplified expression equal to a given area and solve the resulting linear equation?
- Can they use a value they have just solved for in a follow-up calculation, such as a perimeter?