Key focus

This LO covers five core circle theorems: the tangent-radius angle is always $90°$; the angle in a semicircle is always $90°$; angles in the same segment (subtending the same arc, on the same side of the chord) are equal; the angle at the centre is double the angle at the circumference on the same arc; and opposite angles in a cyclic quadrilateral sum to $180°$. Students learn each theorem individually before tackling multi-step problems that chain two theorems together.

Suggested teaching sequence

1. Introduce the tangent-radius theorem: a tangent touches a circle at exactly one point, and the radius to that point is always perpendicular to the tangent.
2. Introduce the angle-in-a-semicircle theorem: any angle drawn from the two ends of a diameter to a point on the circumference is $90°$.
3. Introduce the same-segment theorem: angles subtending the same arc, from any point on the major arc, are equal.
4. Teach the angle-at-the-centre and same-segment theorems together, since both describe angles subtending the same arc. Use the isosceles triangle formed by two radii (equal base angles) to justify why the centre angle is exactly double the circumference angle.
5. Introduce the cyclic quadrilateral theorem, and show it follows from the centre/circumference theorem: each pair of opposite angles subtends the reflex and non-reflex angle at the centre from the same arc, so they must sum to $180°$.
6. Practise multi-step problems that chain two theorems (e.g. tangent-radius then angle-sum in a triangle, or centre/circumference then cyclic quadrilateral), with students labelling which theorem justifies each step.
7. Flag the extension theorems beyond this LO's core five for stronger students: the perpendicular from the centre to a chord bisects the chord (a Pythagoras application), the alternate segment theorem, and intersecting chord/secant theorems.

Core examples to model

- Tangent-radius: a tangent touches a circle at T; the angle between the tangent and radius OT is $90°$.
- Semicircle: an angle in a semicircle, subtended from a diameter, is always $90°$, regardless of where the third point sits on the circumference.
- Same segment: if angles at C and D both subtend chord AB from the same side, and the angle at C is $53°$, the angle at D is also $53°$.
- Centre/circumference: if the angle at the centre is $80°$, the angle at the circumference on the same arc is $40°$ (half).
- Cyclic quadrilateral: if one angle is $102°$, the opposite angle is $78°$ (since $102 + 78 = 180$).
- Multi-step: a tangent-radius right angle combined with a known angle in the same triangle

lets you find a third angle by angle-sum, which can then feed into a same-segment or cyclic-quadrilateral step elsewhere in the diagram.

Watch for

- Confusing the centre angle and circumference angle direction: the centre angle is *double* the circumference angle, not half – check which one the question gives before doubling or halving.
- Assuming any two angles in a circle are equal without checking they subtend the same arc, from the same side of the chord.
- Forgetting that opposite angles in a cyclic quadrilateral means the pair diagonally across the shape, not adjacent angles next to each other – and that *both* diagonal pairs sum to $180°$ independently, not just the pair mentioned in the question.
- Applying the semicircle theorem when the chord is not actually a diameter – the $90°$ result only holds when the two given points are the endpoints of a diameter.
- In multi-step problems, not labelling which theorem justifies each step, which makes an arithmetic slip hard to trace back.
- Treating an underdetermined problem (not enough independent equations for the number of unknowns) as if it has a unique answer – always check that the given information pins down every unknown.

Checks for understanding

- Can students state all five core theorems accurately, including which angle is doubled and which pair must sum to $180°$?
- Can they identify which theorem applies from a labelled diagram or a written description?
- Can they set up and solve an algebraic equation from a theorem relationship (e.g. two expressions that must be equal, or must sum to a fixed total)?
- Can they chain two theorems together in a multi-step problem, explaining which theorem justifies each step?
- Can they explain *why* a theorem is true (e.g. using the isosceles radii triangle for the centre/-circumference theorem), not just apply it mechanically?
- Do they know that bearings, triangle construction, and the extension circle theorems (perpendicular bisecting a chord, alternate segment, intersecting chords/secants) sit beyond this LO's core five?