Teacher guidance

Key focus

This objective focuses on the converse of Pythagoras' theorem: given three side lengths, students determine whether they form a right-angled triangle by testing whether $a^2 + b^2 = c^2$. The key skill is identifying the longest side first (which must be tested as the potential hypotenuse), squaring all three sides correctly, and comparing the exact values — approximations or close matches do not count.

Suggested teaching sequence

1. Revisit $a^2 + b^2 = c^2$ and confirm that the longest side is the hypotenuse.
2. Introduce the test using well-known Pythagorean triples: check $(3, 4, 5)$, $(5, 12, 13)$, $(7, 24, 25)$ by computing both sides and comparing.
3. Emphasise identifying the longest side first; model checking $(5, 12, 13)$ vs $(12, 5, 13)$ to show the order matters.
4. Practise with non-right-angled examples: check $(6, 8, 9)$ and $(8, 15, 16)$ to show that closeness is not enough.
5. Introduce scaled Pythagorean triples: $(6, 8, 10)$ is a $2\times$ scale of $(3, 4, 5)$, so it is right-angled.
6. Move to problems asking students to show the square check in writing: $3^2 + 4^2 = 9 + 16 = 25 = 5^2$, so it is right-angled.
7. Introduce finding a missing side: given two sides and the knowledge that the triangle is right-angled, use $a^2 + b^2 = c^2$ to find the third.
8. Move to reasoning problems: explain why a triangle is or is not right-angled, or compare two triangles using the converse test.

Core examples to model

- Simple triple: check $(3, 4, 5)$. $3^2 + 4^2 = 9 + 16 = 25$ and $5^2 = 25$, so yes, it is right-angled.
- Non-triple but close: check $(8, 15, 16)$. $8^2 + 15^2 = 64 + 225 = 289$ and $16^2 = 256$. Since $289 \neq 256$, it is not right-angled, despite being close.
- Scaled triple: check $(9, 12, 15)$. $9^2 + 12^2 = 81 + 144 = 225$ and $15^2 = 225$, so yes, it is right-angled (it is $3\times$ the $(3, 4, 5)$ triple).
- Finding a missing side: if sides $(x, 24, 25)$ form a right-angled triangle, then $x^2 + 24^2 = 25^2$, so $x^2 = 625 - 576 = 49$ and $x = 7$.
- Comparison: which is closer to right-angled, $(8, 15, 16)$ or $(8, 15, 17)$? For $(8, 15, 17)$: $8^2 + 15^2 = 289$ and $17^2 = 289$, so it is exactly right-angled. $(8, 15, 17)$ is the right-angled one.

Watch for

- Forgetting to identify the longest side first: students must test the longest side as the hypotenuse, not just any side.
- Arithmetic errors when squaring or adding: $8^2 + 15^2$ is often computed as $64 + 225$ but carelessly written as $289 = 17^2$ when in fact $289 = 17^2$ is correct but the student meant $16^2 = 256 \neq 289$.
- Accepting a close match as correct: students must check for exact equality, not approximation. $(8, 15, 16)$ is close but not right-angled.
- Misidentifying scaled triples: emphasise that a scaled triple is still right-angled. If $(3, 4, 5)$ is right-angled, then so is $(6, 8, 10)$, $(9, 12, 15)$, etc.
- Forgetting which side must be tested as the hypotenuse when finding a missing side: if you are told the triangle is right-angled and given two sides, the unknown is either a leg or the hypotenuse depending on which is longest.
- Incorrect reasoning in comparison problems: students must compare the actual squared values, not just state which triangle looks more right-angled.

Checks for understanding

- Can students identify the longest side before applying the test?
- Can they compute a^2, b^2, and c^2 correctly and compare for exact equality?
- Can they distinguish between a right-angled triangle and a close approximation?
- Can they recognise and explain why scaled Pythagorean triples remain right-angled?
- Can they find a missing side given two sides and the knowledge that the triangle is right-angled?
- Can they justify their reasoning in word-based or comparison problems?