## Key focus

This objective combines Pythagoras’ theorem and trigonometric ratios (sine, cosine, tangent) to solve multi-step problems in three-dimensional contexts. Students learn to identify when a problem requires both tools: use Pythagoras’ theorem to find a hidden face or space diagonal, then apply trigonometry to find an angle or height. The key is recognising which right-angled triangle to extract from the 3D shape and ensuring the angle is interpreted correctly (from the base or from the vertical).

## Suggested teaching sequence

1. Review Pythagoras’ theorem and its application to finding face and space diagonals in 3D shapes (see “Applying Pythagoras’ theory to 3D representations”).
2. Review trigonometric ratios in 2D: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.
3. Introduce angle measurement in 3D contexts: when a sloping line (rod, support, cable) goes from the floor to a height, the angle to the base is the angle between the sloping line and its horizontal projection on the ground.
4. Model two-step solutions:
   - *Step-then-trigonometry*: Find a face diagonal or space diagonal using Pythagoras, then use that result with trigonometry.
   - *Trigonometry-then-step*: Find a height or horizontal distance using trigonometry, then use Pythagoras to find another measurement.
5. Show how to extract the correct right-angled triangle from a 3D shape: a sloping support line, a face diagonal, and the edges they connect must form a right angle before applying trigonometry.
6. Emphasize angle direction: if the angle is “to the base,” the base edge is adjacent to the angle; if the angle is “to the vertical,” the role of opposite and adjacent switches.
7. Practice identifying which triangle to use when multiple triangles appear in the same problem (e.g. a cuboid has many possible right triangles).
8. Apply to contextual word problems: ladders against walls, roof struts, mast supports, cables in structures, shipping container problems, etc.
9. Discuss rounding and exact answers: when an intermediate Pythagoras step yields an irrational result, students should carry exact values forward until the final trigonometric step to avoid compounding rounding error.

## Core examples to model

- Base diagonal and space diagonal, then trigonometry: A cuboid has base $6$ cm by $8$ cm (base diagonal $10$ cm) and height $24$ cm. Space diagonal: $D = \sqrt{10^2 + 24^2} = 26$ cm. If this space diagonal makes an angle with the base, $\sin\theta = \dfrac{24}{26} \approx 0.923$, so $\theta \approx 67.4°$.

- Direct trigonometry from a sloping line: A support line is $18$ m long and rises at $50°$ to the base. Height = $18\sin 50° \approx 13.8$ m.
- Combined steps: A cuboid is $8$ cm by $6$ cm by $24$ cm. A line goes from one corner to the opposite corner (space diagonal $26$ cm). Find the angle this line makes with the base: $\sin\theta = \dfrac{24}{26}$, so $\theta \approx 67.4°$.
- Inverse problem: A line inside a box has length $26$ cm and makes angle $22°$ with the base. The vertical height is $26\sin 22° \approx 9.7$ cm. The horizontal floor distance is $26\cos 22° \approx 24.1$ cm.
- Pythagorean triple + trigonometry: A cuboid is $3$ cm by $4$ cm by $12$ cm (space diagonal $13$ cm). The angle to the base: $\sin\theta = \dfrac{12}{13}$, so $\theta = \arcsin\left(\dfrac{12}{13}\right) \approx 67.4°$.
- Two-step forward problem: A ladder leans against a wall inside a frame. The frame is $9$ m wide and $12$ m high; its diagonal across that face is $15$ m. A support line runs from floor to that diagonal at $35°$ angle. Find the height reached: first identify the right triangle, then use $h = \text{slop length} \times \sin 35°$.

## Watch for

- Mixing angle directions: if the problem says "angle to the base," the base distance is adjacent and the height is opposite. Misreading this swaps sine and cosine.
- Forgetting the first Pythagoras step: some problems require finding a face or space diagonal before any trigonometry is possible; jumping straight to trigonometry with incomplete information gives a wrong answer.
- Rounding too early: if a face diagonal is irrational (e.g. $\sqrt{481}$), using a rounded value in the next trigonometric step can shift the final angle or height significantly.
- Confusing the 3D angle with a 2D angle: the angle a space diagonal makes with the base is not the same as the angle on any single rectangular face; it is the angle between the diagonal and its horizontal projection.
- Using the wrong triangle: a 3D problem may display multiple right triangles (one on each face); students must select the triangle that includes the quantity they are solving for.
- Forgetting that a line can be the hypotenuse or another side: depending on the question, a sloping support might be the hypotenuse (if given length and angle), the opposite side (if given height and angle), or neither.
- Assuming an exact trigonometric value (e.g. $\sin 30° = 0.5$) when the angle is not a special angle; use a calculator for non-standard angles and round the final answer as requested.

## Checks for understanding

- Can students find a face diagonal or space diagonal using Pythagoras' theorem and then extract the correct right-angled triangle for trigonometry?
- Can they apply sine, cosine, or tangent to find a height, horizontal distance, or angle in a 3D context?
- Can they identify whether an angle is measured from the base or the vertical and apply the correct ratio?

- Can they solve multi-step problems that require both Pythagoras' theorem and trigonometry in sequence?
- Can they justify (in words or with a sketch) why a particular trigonometric ratio is the right choice for a given problem?
- Can they work with exact values (surds or Pythagorean triples) in intermediate steps and round only the final answer?
- Can they solve inverse problems: given a space diagonal and an angle, find a height; or given a height and an angle, find the sloping length?
- Can they apply these techniques to real-world contexts such as ladders, roof struts, mast supports, and shipping containers?