Key focus

This objective combines two skill strands rather than teaching a new geometry fact: reading a labelled diagram, writing an algebraic expression from a geometric fact students already know (angle rules, perimeter/area formulas, polygon interior-angle sums, similar-shape scale factors), then solving the resulting linear or quadratic equation. Angle-side questions draw on straight-line, vertically-opposite, corresponding, co-interior, triangle-angle-sum, and regular-polygon interior-angle facts – students should identify which angle fact applies before writing the equation. Shape-side questions draw on perimeter and area formulas (rectangle, square, triangle, trapezium, and simple composite L-shapes) plus similar-shape scale factors; area problems can produce a quadratic equation (e.g. $x(x+4) = 60$), so only the positive root usually fits a length.

Suggested teaching sequence

1. Revisit the angle facts needed: angles on a straight line sum to $180°$, vertically opposite angles are equal, corresponding angles are equal, co-interior angles sum to $180°$, and angles in a triangle sum to $180°$.
2. Practise the two-step method on one-step angle problems: state the fact as an equation, then solve for x.
3. Extend to two-step linear equations where both angles are algebraic expressions (e.g. two co-interior angles both containing x).
4. Introduce regular-polygon interior-angle problems: the polygon's known interior angle (e.g. $108°$ for a pentagon, $120°$ for a hexagon) is set equal to the algebraic expression.
5. Move to shape-side perimeter and area problems: rectangle and square perimeter, triangle area, then pictured trapeziums and simple L-shapes, each rearranged into an equation in x.
6. Introduce similar-shape scale-factor problems, written either as a direct scale-factor multiplication or as a proportion between two pairs of corresponding sides.
7. Introduce area problems that produce a quadratic equation, such as $x(x+4) = 60$ or $\frac{1}{2}((x+1)+(x+5))x = 54$. Model factorising (or the quadratic formula when it does not factorise neatly), finding both roots, and rejecting the root that cannot be a length.
8. Combine skills in multi-step problems: find x from one geometric fact, then use it for a second calculation such as a perimeter, a height, or an adjacent angle.

Core examples to model

- Straight-line angle fact: angles on a straight line are $(x+20)°$ and $70°$. $(x+20)+70 = 180$, so $x = 90$.
- Two-step linear equation: co-interior angles are $(2x+20)°$ and $(x+40)°$. $(2x+20)+(x+40) = 180$, so $3x+60 = 180$ and $x = 40$.
- Regular polygon: a regular pentagon has interior angle $108°$; an angle is labelled $(3x+6)°$. $3x+6 = 108$, so $x = 34$.
- Perimeter fact: a rectangle has length 6 cm and width x cm with perimeter 22 cm.

$2(6+x)=22$, so $x=5$.

- Composite-area fact: an L-shape is split into a bottom rectangle $4(2x+2)$ and a top-left rectangle $2(x-4)$. If the total area is 50 cm^2, then $4(2x+2)+2(x-4)=50$, so $10x=50$ and $x=5$.
- Trapezium-area fact: a trapezium has parallel sides $(x+2)$ cm and 10 cm, height 4 cm, and area 28 cm^2. $\frac{1}{2}((x+2)+10)4=28$, so $2x+24=28$ and $x=2$.
- Quadratic area fact: a rectangle has width x cm and length $(x+4)$ cm with area 60 cm^2. $x(x+4)=60$ gives $x^2+4x-60=0$, which factorises to $(x+10)(x-6)=0$, so $x=6$ or $x=-10$; reject $x=-10$ since a length cannot be negative.
- Similar shapes: two similar rectangles have corresponding sides 6 cm and 9 cm; the smaller rectangle's other side is x cm and the larger's is 12 cm. $\dfrac{x}{12}=\dfrac{6}{9}$, so $x=8$.
- Multi-step problem: a rectangle has width x cm and length $(x+3)$ cm with area 70 cm^2. Solving $x(x+3)=70$ gives $x=7$ (rejecting $x=-10$); the perimeter is then $2(7+10)=34$ cm.

Watch for

- Using the wrong angle fact: confusing an "equal" relationship (vertically opposite, corresponding, alternate) with a "sums to $180°$" relationship (angles on a straight line, co-interior) gives the wrong equation even when the algebra afterwards is done correctly.
- Forgetting to distribute correctly in a perimeter equation, e.g. writing $2(x+3)=2x+3$ instead of $2x+6$.
- Reading the diagram too loosely: on an L-shape, students may add the labelled side lengths correctly but forget that the area still comes from splitting the shape into rectangles, not from multiplying unrelated labels together.
- Using the trapezium formula with the non-parallel sides, or forgetting the one-half factor in $A=\frac{1}{2}(a+b)h$.
- Reporting both roots of a quadratic area equation without checking which one makes sense – a negative or zero length must be rejected.
- Stopping one step too early on a multi-step problem: after finding x, check whether the question also asks for a perimeter, an area, a height, or another angle.
- Setting a regular-polygon equation equal to the wrong angle sum – interior and exterior angles are different quantities, and only one is usually given.
- Losing track of which side of a similar-shape proportion is which – matching corresponding sides on the same side of the equation (small over small, large over large, or vice versa consistently) avoids an inverted ratio.

Checks for understanding

- Can students identify which angle fact applies (equal vs. supplementary) before writing the equation?
- Can they write and solve a two-step linear equation from a perimeter, area, or angle fact?
- Can they split a pictured composite shape into simpler rectangles and turn the diagram into

one clean area equation?

- Can they identify the parallel sides and perpendicular height on a pictured trapezium before substituting into the formula?
- Can they set up and solve a regular-polygon interior-angle equation?
- Can they recognise when an area problem produces a quadratic equation, solve it, and reject the root that does not make sense as a length?
- Can they set up and solve a similar-shape scale-factor proportion?
- Can they use a first answer as the input to a second calculation in a multi-step problem?