Answers — apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons.

1. Perimeter 56, two sides 15 and 20. Could it be right-angled?

Solution

Third side $= 56 - 15 - 20 = 21$.
$15^2 + 20^2 = 225 + 400 = 625 \neq 441 = 21^2$.
No.

2. Perimeter 70, two sides 21 and 29. Could it be right-angled?

Solution

Third side $= 70 - 21 - 29 = 20$.
$20^2 + 21^2 = 400 + 441 = 841 = 29^2$.
Yes.

3. Side lengths $3k, 4k, 5k$. Prove it is always right-angled for positive k.

Solution

$(3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2 = (5k)^2$ for every positive k.

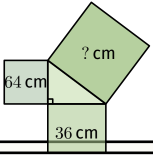
4. One side is 25. Find two other whole-number sides so it is right-angled, and prove your triple works.

Solution

$7, 24, 25$: $7^2 + 24^2 = 625 = 25^2$.

Answers — apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons.

5. Hypotenuse square: area and side length?



Solution

$36 + 64 = 100.$

Area $= 100$, side $= 10.$

6. A student checks $9^2 + 40^2 = 41^2$ and says that proves right-angled. What assumption did they make?

Solution

That 41 is the hypotenuse (longest side).

7. Which is closer to being right-angled: $(8, 15, 16)$ or $(8, 15, 18)$? Justify using $a^2 + b^2 - c^2$.

Solution

$8^2 + 15^2 = 289.$

$289 - 16^2 = 33;\ 289 - 18^2 = -35.$

$|33| < 35$, so $(8, 15, 16)$ is closer.

8. Which is closer to being right-angled: $(10, 24, 25)$ or $(10, 24, 27)$? Justify using $a^2 + b^2 - c^2$.

Solution

$10^2 + 24^2 = 676.$

$676 - 25^2 = 51;\ 676 - 27^2 = -53.$

$|51| < 53$, so $(10, 24, 25)$ is closer.

Answers — apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons.

9. A triangle has sides $x, 35, 37$. Find x if the triangle is right-angled.

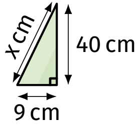
Solution

37 is the hypotenuse.

$x^2 = 37^2 - 35^2 = 1369 - 1225 = 144$

$x = 12$.

10. Right-angled. Find x; explain how you knew the hypotenuse first.



Solution

Opposite the right angle $\Rightarrow$ hyp.

$x^2 = 1681, \ x = 41$.

11. A student says every triangle with sides in the ratio $5 : 12 : 13$ is right-angled. Correct? Explain using the scaling property.

Solution

Yes — $(5k)^2 + (12k)^2 = (13k)^2$ for all k.

12. Create a whole-number triangle close to $(9, 40, 41)$ that is not right-angled, and explain why it fails.

Solution

Example: $9, 40, 42$.

$9^2 + 40^2 = 1681 \neq 1764 = 42^2$, so it fails the check.