

Simplifying fractions with unfactorised quadratic in denominator or numerator



Notes & Steps

Key idea

When a fraction has an unfactorised quadratic in the numerator or denominator, factorise first. Only then can you cancel common *factors*. You cannot cancel parts of a sum or difference.

Steps

1. Factorise the numerator completely.
2. Factorise the denominator completely.
3. Identify any common factor in the whole numerator and denominator.
4. Cancel the common factor.
5. State any restrictions from the original denominator.

Common mistake

Cancelling terms instead of factors. In $\frac{x^2 - 9}{x^2 - 3x}$, you cannot cancel the x^2 . You must first factorise to $\frac{(x - 3)(x + 3)}{x(x - 3)}$.

Reminders

- ▶ $(x^2 - 9) = (x - 3)(x + 3)$
- ▶ $(x^2 + 5x) = x(x + 5)$
- ▶ Restrictions come from the *original* denominator.
- ▶ If there is no common factor, leave the fraction as it is.



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Example 1: quadratic numerator

Simplify $\frac{x^2 + 5x}{x}$.

$$x^2 + 5x = x(x + 5)$$

$$\frac{x(x + 5)}{x} = x + 5$$

Restriction: $x \neq 0$.

Example 2: difference of squares

Simplify $\frac{x^2 - 9}{x - 3}$.

$$x^2 - 9 = (x - 3)(x + 3)$$

$$\frac{(x - 3)(x + 3)}{x - 3} = x + 3$$

Restriction: $x \neq 3$.

Try these

1. Simplify $\frac{x^2 - 16}{x - 4}$.

2. Simplify $\frac{x^2 + 8x + 12}{x + 2}$.

3. Simplify $\frac{x^2 - 25}{x^2 + 5x}$.

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Example 3: both numerator and denominator factorise

Simplify $\frac{x^2 + 5x + 6}{x^2 + 2x}$.

$$\frac{(x+2)(x+3)}{x(x+2)}$$

Cancel the common factor $(x+2)$:

$$\frac{x+3}{x}$$

Restrictions: $x \neq 0, -2$.

Example 4: no common factor left

Simplify $\frac{45x^2 + 72x - 5}{30x^2 - 29x + 6}$.

$$\frac{(15x-1)(3x+5)}{(10x-3)(3x-2)}$$

There is no common factor, so this is already simplified.

Common mistake

Forgetting the restriction after a factor cancels. Even if $(x-3)$ cancels, $x=3$ still makes the original denominator zero, so it must stay excluded.