Multiplying expressions with powers

This objective develops the product law for powers with the same base.

Vocabulary

A **variable** is a letter standing for a number. An **exponent** (or **power/index**) tells how many equal factors are multiplied. A **product** is the result of multiplication. To **simplify** is to write an equivalent expression clearly. The **coefficient** is the numerical factor.

The key idea

When multiplying powers with the same base, $a^m \times a^n = a^{m+n}$. Keep the base; add the exponents.

Why the exponents add

Expand short products to see the structure.

Repeated multiplication

$x^2 \times x^3 = (x \times x)(x \times x \times x) = x^5$. There are five factors of x, so $2 + 3 = 5$.

Numerical bases

$2^3 \times 2^4 = 2^{3+4} = 2^7$. Do not multiply the exponents.

Common mistake

$x^2 \times x^3 \neq x^6$. Multiplying exponents is for a different law, introduced later.

A reliable process

Coefficients and powers are simplified separately.

Worked example

$4a^2 \times 3a^5 = (4 \times 3)(a^2 \times a^5) = 12a^{2+5} = 12a^7$.

With two variables

$2x^3y \times 5x^2y^4 = 10x^{3+2}y^{1+4} = 10x^5y^5$. Only equal bases combine.

Check

For $(-3)m^2 \times 4m^3$, multiply the coefficients first: -12, then use m^{2+3}, giving $-12m^5$.

Products in area

Area gives a useful algebraic multiplication model.

Square

A square with side $3y^4$ has area $(3y^4)(3y^4) = 9y^8$.

Rectangle

A $4x^2y$ by $3xy^4$ rectangle has area $12x^3y^5$.

Reasoning

A negative dimension is not a physical measurement, but it is still useful algebra practice: $(-2k^3)(-2k^3) = 4k^6$.

Checks and boundaries

Use the product law accurately and know what comes next.

Three checks

1. Multiply coefficients. 2. Add exponents only for identical bases. 3. Keep different variables separate.

Error analysis

$3x^2 \times 2x^4 = 6x^6$, not $5x^6$ (coefficients multiply) and not $6x^8$ (exponents add).

Scope

This objective is multiplication of powers. Dividing powers belongs to LO10; powers of powers such as $(x^3)^2$ are deliberately deferred.