Limits - Notes and Steps

Key idea

A **limit** asks what value $f(x)$ heads towards as x gets close to a.

Written $\lim\limits_{x \to a} f(x)$.

First try: substitute

Put $x = a$ in. A number means that is the limit.

$\lim\limits_{x \to 2}(x + 3) = 5$

Common mistake

Substitution fails when the bottom becomes 0, giving $\frac{0}{0}$.

$\frac{0}{0}$ is **not** 0 or 1 — it means "try another way" (next slides).

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When You Get 0/0: Factorise

Key idea

If substitution gives $\dfrac{0}{0}$:

factorise top and bottom, **cancel** the common factor, then substitute again.

Example

$$\lim_{x\to 5}\frac{x^2-25}{x-5}$$
$$=\lim_{x\to 5}\frac{(x-5)(x+5)}{x-5}$$
$$=\lim_{x\to 5}(x+5)=10$$

Common mistake

Giving up at $\dfrac{0}{0}$.

Cancel the $(x-5)$ first, then put $x=5$.

Limits at Infinity

Key idea

For $x \to \infty$, divide every term by the **highest power** of x.

Then $\frac{1}{x}, \frac{1}{x^2} \to 0$.

Example

$$\lim_{x\to\infty} \frac{3x^2 + 2x}{5x^2 - x + 1}$$

Divide by x^2: $\dfrac{3 + \frac{2}{x}}{5 - \frac{1}{x} + \frac{1}{x^2}} \to \dfrac{3}{5}$

Common mistake

Compare the **leading** coefficients: the answer is $\frac{3}{5}$, not 3 or 5.

The h to 0 Form

Key idea

For $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$:
expand and **simplify first**, then let $h \to 0$.

Example

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{2xh + h^2}{h} = 2x + h$$

As $h \to 0$: $= 2x$

Common mistake

Putting $h = 0$ too early gives $\dfrac{0}{0}$.
Cancel the h first, then let $h \to 0$.

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