The Power Rule

Key idea

If $f(x) = x^n$ then
$f'(x) = nx^{n-1}$.

Multiply by the exponent, then drop the exponent by one.

With a coefficient

If $f(x) = ax^n$ then $f'(x) = anx^{n-1}$.

$f(x) = 5x^4 \;\rightarrow\; f'(x) = 20x^3$

Common mistake

Multiply by the coefficient too.

For $f(x) = 3x^4$: $f'(x) = 3 \times 4 \times x^3 = 12x^3$, **not** $4x^3$.

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Differentiate Term by Term

Key idea

Differentiation is linear: differentiate each term on its own and add the results.

A constant differentiates to 0.

Example

$f(x) = 3x^4 + 6x^3 - 7x^2 - 2x + 3$

$f'(x) = 12x^3 + 18x^2 - 14x - 2$

(The $+3$ differentiates to 0.)

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Rewrite as Powers First

Key idea

Write fractions and roots as powers of x first, then use the power rule:

$$\frac{1}{x} = x^{-1} \qquad \frac{1}{x^2} = x^{-2}$$

$$\sqrt{x} = x^{\frac{1}{2}}$$

Fractions

$$f(x) = 3x^2 + \frac{2}{x} + 5$$

$$= 3x^2 + 2x^{-1} + 5$$

$$f'(x) = 6x - 2x^{-2} = 6x - \frac{2}{x^2}$$

Roots

$$f(x) = 4\sqrt{x} = 4x^{\frac{1}{2}}$$

$$f'(x) = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$$

Simplify First: Products & Quotients

Expand first

$f(x) = (2x - 1)(x + 3)$

$= 2x^2 + 5x - 3$

$f'(x) = 4x + 5$

Divide through first

$$f(x) = \frac{4x^3 + 7x^2 - 6x - 5}{x^2}$$

$$= 4x + 7 - 6x^{-1} - 5x^{-2}$$

$$f'(x) = 4 + \frac{6}{x^2} + \frac{10}{x^3}$$

Common mistake

Divide *every* term by the denominator: $(x^3 + 2x)/x = x^2 + 2$, **not** $x^3 + 2$.

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