

Deriving from First Principles



Key idea

The derivative $f'(x)$ is the **gradient function**, found from first principles:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Method

1. Write $f(x+h)$.
2. Simplify $f(x+h) - f(x)$.
3. Divide by h .
4. Let $h \rightarrow 0$; the h terms vanish.

Common mistake

Expand in full: $(x+h)^2 = x^2 + 2xh + h^2$, **not** $x^2 + h^2$.



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First Principles - Worked Examples



Example: $f(x) = x^2$



$$f(x+h) = x^2 + 2xh + h^2$$

$$f(x+h) - f(x) = 2xh + h^2$$

$$\frac{2xh + h^2}{h} = 2x + h$$

$$\text{As } h \rightarrow 0: f'(x) = 2x$$

Example: $f(x) = 3x^2 + 2x$



$$f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$$

$$f(x+h) - f(x) = 6xh + 3h^2 + 2h$$

$$\frac{h(6x + 3h + 2)}{h} = 6x + 3h + 2$$

$$\text{As } h \rightarrow 0: f'(x) = 6x + 2$$

Common mistake

Keep every term with an h until the last step — only then let $h \rightarrow 0$.



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