Volume of pyramids, cones, and spheres

Mana Maths

Te reo Māori terms

koeko
cone
Open in Te Aka

rorahi
volume
Open in Te Aka

koeko hotiu
pyramid
Open in Te Aka

ahutoru
solid shape
Open in Te Aka

Cones and Pyramids - Volume

Key idea

A cone or pyramid holds **one third** of the prism on the same base:

$V = \frac{1}{3} \times$ base area $\times h$.

Use the **perpendicular** height h.

Cone

Radius $= 3$, height $= 8$.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3^2)(8) = 24\pi$

Square pyramid

Base side $= 10$, height $= 9$.

Base area $= 10^2 = 100$.

$V = \frac{1}{3}(100)(9) = 300$

Common mistake

Do not use the **slant** height in a volume formula.

Volume needs the straight-up perpendicular height.

Spheres - Volume

Key idea

A sphere's volume depends only on its radius:
$V = \frac{4}{3}\pi r^3$.
Keep π exact unless a decimal is asked for.

Example

Radius $= 3$.
$V = \frac{4}{3}\pi(3^3) = \frac{4}{3}\pi(27) = 36\pi$

Common mistake

Cube the radius first, then multiply.
r^3 means $r \times r \times r$, so $3^3 = 27$ — not $3 \times 3 = 9$.

Start Tasks

1. A cone has radius 3 cm and height 8 cm. Find its volume in terms of π.

2. A cone has radius 5 m and height 6 m. Find its volume in terms of π.

3. A sphere has radius 3 cm. Find its volume in terms of π.

4. A sphere has radius 6 m. Find its volume in terms of π.

5. A square pyramid has base side 10 cm and perpendicular height 9 cm. Find its volume.

6. A square pyramid has base side 12 m and perpendicular height 5 m. Find its volume.

7. A rectangular pyramid has base 8 cm by 5 cm and height 9 cm. Find its volume.

8. A rectangular pyramid has base 9 m by 4 m and height 7 m. Find its volume.

9. A cone has radius 6 cm and height 9 cm. Find its volume in terms of π.

Start Tasks Answers

1. 24π cm^3

2. 50π m^3

3. 36π cm^3

4. 288π m^3

5. 300 cm^3

6. 240 m^3

7. 120 cm^3

8. 84 m^3

9. 108π cm^3

Start Tasks

10. A cone has volume 48π cm^3 and height 4 cm. Find the radius.

11. A sphere has volume $500\pi/3$ cm^3. Find the radius.

12. A square pyramid has volume 144 cm^3 and base side 6 cm. Find the height.

13. A cone has radius 4 cm. If the height doubles, what happens to the volume?

14. A sphere has radius 5 cm. If the radius doubles, is the volume doubled, quadrupled, or multiplied by 8?

15. A square pyramid has base side 9 cm and height 6 cm. Find its volume.

16. A cone has radius 2 m and height 12 m. Find its volume in terms of π.

17. A sphere has radius 2.5 cm. Find its volume correct to 1 d.p.

18. A rectangular pyramid has base 10 cm by 6 cm and height 8 cm. Find its volume.

Start Tasks Answers

10. 6 cm

11. 5 cm

12. 12 cm

13. It doubles.

14. Multiplied by 8.

15. 162 cm^3

16. 16π m^3

17. 65.4 cm^3

18. 160 cm^3

Start Tasks

19. A cone has radius 7 cm and height 3 cm. Find its volume in terms of π.

20. A sphere has radius 4 m. Find its volume in terms of π.

21. A square pyramid has base side 15 cm and height 4 cm. Find its volume.

22. A cone has volume 75π cm^3 and radius 5 cm. Find the height.

23. A square pyramid has volume 392 cm^3 and height 6 cm. Find the base area.

24. A sphere has volume $\frac{4}{3}\pi(4^3)$. What is the radius?

25. Which solid uses the formula $\frac{4}{3}\pi r^3$?

26. Which solid uses one third of base area times height?

27. Why is the cone formula similar to the pyramid formula?

Start Tasks Answers

19. 49π cm^3

20. $\frac{256}{3}\pi$ m^3

21. 300 cm^3

22. 9 cm

23. 196 cm^2

24. 4

25. Sphere.

26. Pyramid and cone.

27. Because both use one third of a full prism/cylinder-style base-times-height volume

Build Tasks

1. A cone has radius 10 cm and height 24 cm. Find its volume in terms of π.

2. A sphere has radius 7 cm. Find its volume in terms of π.

3. A square pyramid has base side 9 cm and height 14 cm. Find its volume.

4. A rectangular pyramid has base 12 m by 7 m and height 10 m. Find its volume.

5. A cone has volume 75π m^3 and radius 5 m. Find its height.

6. A sphere has volume $500\pi/3$ cm^3. Find its radius.

7. A square pyramid has volume 392 cm^3 and height 6 cm. Find the side length of the base.

8. A cone and a cylinder have the same radius 6 cm and the same height 12 cm. Find both volumes and compare.

9. A hemisphere has radius 9 cm. Find its volume in terms of π.

Build Tasks Answers

1. 800π cm^3

2. $\frac{1372}{3}\pi$ cm^3

3. 378 cm^3

4. 280 m^3

5. 9 m

6. 5 cm

7. 14 cm

8. Cone $= 144\pi$, cylinder $= 432\pi$, so cylinder is 3 times as large.

9. 486π cm^3

Build Tasks

10. A square pyramid has volume 250 cm^3 and height 6 cm. Find the side length of the base to 1 d.p.

11. A sphere has radius 4.5 cm. Find its volume correct to 1 d.p.

12. A cone has radius 8 cm and volume 256π cm^3. Find the height.

13. A square pyramid has base side 16 cm and volume 512 cm^3. Find the height.

14. A sphere and a cone both use radius 6 cm. The cone height is 18 cm. Which has larger volume?

15. A cone has radius 9 cm and slant height 15 cm. Find its perpendicular height, then its volume in terms of π.

16. A hemisphere and a cone both have radius 3 cm. The cone height is 8 cm. Find both volumes.

17. A rectangular pyramid has base 15 cm by 8 cm and volume 320 cm^3. Find the height.

18. A sphere has volume 288π cm^3. Form an equation and solve for the radius.

Build Tasks Answers

10. 11.2 cm

11. 381.7 cm^3

12. 12 cm

13. 6 cm

14. Sphere, because $\frac{4}{3}\pi 6^3 = 288\pi$ while the cone is 216π.

15. Height $= 12$ cm, volume $= 324\pi$ cm^3

16. Hemisphere $= 18\pi$, cone $= 24\pi$

17. 8 cm

18. $r^3 = 216$, so $r = 6$

Build Tasks

19. A cone has radius 4 cm. Its volume is the same as a cylinder with the same radius and height 5 cm. Find the cone height.

20. A toy is made from a hemisphere of radius 4 cm joined to a cone of radius 4 cm and height 9 cm. Find the total volume in terms of π.

21. A sphere has the same volume as a cone with radius 6 cm and height 8 cm. Is the sphere radius more or less than 4 cm?

22. Explain why the cone formula has the factor $\frac{1}{3}$.

23. Explain why a hemisphere volume is half a sphere volume.

24. Which dimensions do you need for a rectangular pyramid volume?

25. A square pyramid and a prism have the same base and height. If the prism volume is 540 cm^3, find the pyramid

26. A sphere has radius 3 cm. If the radius doubles, by what factor does the volume change?

27. A cone has radius 5 cm and volume 200π cm^3. Find the height.

Build Tasks Answers

19. 15 cm

20. $\frac{232}{3}\pi$ cm^3

21. Less than 4 cm.

22. Because a cone is one third of a matching cylinder.

23. Because a hemisphere is half of the full sphere.

24. The base dimensions and the perpendicular height.

25. 180 cm^3

26. By a factor of 8.

27. 24 cm

Push Tasks

1. A cone and a cylinder both have radius 6 cm and height 12 cm. Show that the cylinder volume is three times the cone volume.

2. A sphere has the same volume as a cylinder with radius 3 cm and height 16 cm. Form an equation and solve for the sphere radius.

3. A square pyramid has base side 12 cm. Its volume is 576 cm^3. Find the perpendicular height.

4. A cone has volume 128π cm^3 and height 24 cm. Find the radius.

5. A square pyramid has the same base area and height as a prism. The prism volume is 540 cm^3. Find the pyramid volume and justify it.

6. A sphere has radius *r*. Write an expression for the volume of a hemisphere in terms of *r*.

7. A cone has radius 9 cm and slant height 15 cm. Find its perpendicular height, then its volume in terms of π.

8. A solid sphere of radius 6 cm is melted and recast into a cylinder of radius 3 cm. Find the cylinder height.

9. A square pyramid has volume 250 cm^3 and height 6 cm. Find the base side length to 1 d.p.

Push Tasks Answers

1. Cylinder $= 432\pi$, cone $= 144\pi$, so the cylinder is 3 times the cone.

2. $\frac{4}{3}\pi r^3 = 144\pi$, so $r^3 = 108$ and $r = 3\sqrt[3]{4}$

3. 12 cm

4. 4 cm

5. 180 cm^3, because the pyramid is one third of the matching prism.

6. $\frac{2}{3}\pi r^3$

7. Height $= 12$ cm, volume $= 324\pi$ cm^3

8. Height $= 32$ cm

9. 11.2 cm

Push Tasks

10. A hemisphere of radius 4 cm is joined to a cone of radius 4 cm and height 9 cm. Find the total volume in terms of π.

11. A sphere has volume 288π cm^3. Form an equation and solve for the radius.

12. A cone and a pyramid both have volume 180 cm^3. The cone has radius 6 cm. The pyramid has square base side 6 cm. Find the height of each.

13. A cone and a sphere both have radius 5 cm. The cone height is 20 cm. Which solid has the larger volume?

14. A rectangular pyramid has base 12 cm by 10 cm and the same volume as a cone of radius 6 cm and height 10 cm. Find the pyramid height.

15. A sphere has the same volume as a cone of radius 4 cm and height 16 cm. Estimate the sphere radius to 1 d.p.

16. A square pyramid has side x, height 12, and volume 400. Find x.

17. A cone has radius r and height h. Write an expression for a cylinder with the same radius and three times the cone volume.

18. A cone has radius 12 cm and volume 384π cm^3. Find the height.

Push Tasks Answers

10. $\frac{232}{3}\pi$ cm^3

11. $r^3 = 216$, so $r = 6$

12. Cone height $= 5$ cm, pyramid height $= 15$ cm

13. Sphere, because it has $\frac{4}{3}\pi 5^3 = \frac{500}{3}\pi$ while the cone has $\frac{1}{3}\pi 5^2(20) = \frac{500}{3}\pi$; they are equal

14. Set $\frac{1}{3}(120)h = 120\pi$, so $h = 3\pi$ cm

15. $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi(4^2)(16)$, so $r^3 = 64$ and $r = 4$

16. $x^2 = 100$, so $x = 10$

17. A cylinder with the same radius and volume $3 \times \frac{1}{3}\pi r^2 h$ has volume $\pi r^2 h$, so its height is h.

18. 8 cm

Push Tasks

19. A cylinder and a sphere both have radius 6 cm. The cylinder height is 8 cm. Which has greater volume, and by how much in terms of π?

20. A cone and a pyramid each have base area 100 cm^2 and height 9 cm. Compare their volumes.

21. A hemisphere has volume 144π cm^3. Form an equation and solve for the radius.

22. Explain why doubling the radius of a sphere changes the volume by a factor of 8.

23. Explain why a pyramid and cone can share the same one-third structure in their formulas.

24. Explain the difference between slant height and perpendicular height in cone and pyramid volume problems.

25. A sphere of radius 3 cm is scaled so every length is multiplied by 2. What is the new volume factor?

26. A cone has radius 5 cm and volume 200π cm^3. If the radius doubles but the volume stays the same, what

27. Write one sentence explaining how cone, pyramid, and sphere volume formulas are connected by area and

Push Tasks Answers

19. Sphere $= 288\pi$, cylinder $= 288\pi$, difference $= 0$.

20. They are equal, because both are one third of base area times height.

21. $\frac{2}{3}\pi r^3 = 144\pi$, so $r^3 = 216$ and $r = 6$

22. Because volume scales with r^3, and $2^3 = 8$.

23. Both are one-third of a matching prism or cylinder with the same base and height.

24. Volume needs the perpendicular height; slant height only helps if you first convert it.

25. By a factor of 8.

26. The height must become one quarter of the original.

27. Cone and pyramid volumes both come from one-third of base-area-times-height, while sphere volume