



Simplifying fractions with unfactorised quadratic in denominator or numerator

Mana Maths

Te reo Māori terms



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Simplifying fractions with unfactorised quadratic in denominator or numerator



Notes & Steps

Key idea

When a fraction has an unfactorised quadratic in the numerator or denominator, factorise first. Only then can you cancel common *factors*. You cannot cancel parts of a sum or difference.

Steps

1. Factorise the numerator completely.
2. Factorise the denominator completely.
3. Identify any common factor in the whole numerator and denominator.
4. Cancel the common factor.
5. State any restrictions from the original denominator.

Common mistake

Cancelling terms instead of factors. In $\frac{x^2 - 9}{x^2 - 3x}$, you cannot cancel the x^2 . You must first factorise to $\frac{(x - 3)(x + 3)}{x(x - 3)}$.

Reminders

- ▶ $(x^2 - 9) = (x - 3)(x + 3)$
- ▶ $(x^2 + 5x) = x(x + 5)$
- ▶ Restrictions come from the *original* denominator.
- ▶ If there is no common factor, leave the fraction as it is.



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Notes & Steps

Example 1: quadratic numerator

Simplify $\frac{x^2 + 5x}{x}$.

$$x^2 + 5x = x(x + 5)$$

$$\frac{x(x + 5)}{x} = x + 5$$

Restriction: $x \neq 0$.

Example 2: difference of squares

Simplify $\frac{x^2 - 9}{x - 3}$.

$$x^2 - 9 = (x - 3)(x + 3)$$

$$\frac{(x - 3)(x + 3)}{x - 3} = x + 3$$

Restriction: $x \neq 3$.

Try these

1. Simplify $\frac{x^2 - 16}{x - 4}$.

2. Simplify $\frac{x^2 + 8x + 12}{x + 2}$.

3. Simplify $\frac{x^2 - 25}{x^2 + 5x}$.

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Notes & Steps

Example 3: both numerator and denominator factorise

Simplify $\frac{x^2 + 5x + 6}{x^2 + 2x}$.

$$\frac{(x+2)(x+3)}{x(x+2)}$$

Cancel the common factor $(x+2)$:

$$\frac{x+3}{x}$$

Restrictions: $x \neq 0, -2$.

Example 4: no common factor left

Simplify $\frac{45x^2 + 72x - 5}{30x^2 - 29x + 6}$.

$$\frac{(15x-1)(3x+5)}{(10x-3)(3x-2)}$$

There is no common factor, so this is already simplified.

Common mistake

Forgetting the restriction after a factor cancels. Even if $(x-3)$ cancels, $x=3$ still makes the original denominator zero, so it must stay excluded.

Start Tasks



1. Simplify $\frac{x^2 + 5x}{x}$.

2. Simplify $\frac{y^2 + 7y}{y}$.

3. Simplify $\frac{a^2 - 4a}{a}$.

4. Simplify $\frac{m^2 - 9m}{m}$.

5. Simplify $\frac{x^2 - 9}{x - 3}$.

6. Simplify $\frac{x^2 - 16}{x - 4}$.

7. Simplify $\frac{y^2 - 25}{y - 5}$.

8. Simplify $\frac{a^2 - 1}{a - 1}$.

9. Simplify $\frac{p^2 + 6p}{p}$.

Start Tasks – Answers



1. $x + 5$

4. $m - 9$

7. $y + 5$

2. $y + 7$

5. $x + 3$

8. $a + 1$

3. $a - 4$

6. $x + 4$

9. $p + 6$

Start Tasks



10. Simplify $\frac{2x^2 + 10x}{2x}$.

11. Simplify $\frac{3y^2 - 6y}{3y}$.

12. Simplify $\frac{4a^2 + 12a}{4a}$.

13. Simplify $\frac{x^2 + 8x + 12}{x + 2}$.

14. Simplify $\frac{x^2 + 9x + 20}{x + 4}$.

15. Simplify $\frac{x^2 - 5x - 6}{x - 6}$.

16. Simplify $\frac{x^2 - 7x + 10}{x - 5}$.

17. Simplify $\frac{y^2 + 3y - 10}{y + 5}$.

18. Simplify $\frac{m^2 - m - 12}{m - 4}$.

Start Tasks – Answers



10. $x + 5$

13. $x + 6$

16. $x - 2$

11. $y - 2$

14. $x + 5$

17. $y - 2$

12. $a + 3$

15. $x + 1$

18. $m + 3$

Start Tasks



19. Simplify $\frac{x^2 + 5x}{x}$ and state any restriction.

20. Simplify $\frac{x^2 - 9}{x - 3}$ and state any restriction.

21. Simplify $\frac{x^2 + 8x + 12}{x + 2}$ and state any restriction.

22. Which factor cancels in $\frac{x^2 - 16}{x - 4}$?

23. Which factor cancels in $\frac{x^2 - 7x + 10}{x - 5}$?

24. Simplify $\frac{q^2 - 4q}{q}$.

25. Simplify $\frac{b^2 + 11b + 30}{b + 5}$.

26. Simplify $\frac{c^2 - 8c + 15}{c - 5}$.

27. Simplify $\frac{r^2 - 36}{r + 6}$.

Start Tasks – Answers



19. $x + 5, x \neq 0$

22. $x - 4$

25. $b + 6$

20. $x + 3, x \neq 3$

23. $x - 5$

26. $c - 3$

21. $x + 6, x \neq -2$

24. $q - 4$

27. $r - 6$

Build Tasks



1. Simplify $\frac{x^2 - 9}{x^2 - 3x}$.

2. Simplify $\frac{x^2 - 16}{x^2 - 4x}$.

3. Simplify $\frac{x^2 - 25}{x^2 + 5x}$.

4. Simplify $\frac{x^2 + 5x + 6}{x^2 + 2x}$.

5. Simplify $\frac{x^2 + 7x + 10}{x^2 + 5x}$.

6. Simplify $\frac{x^2 - 5x + 6}{x^2 - 2x}$.

7. Simplify $\frac{x^2 - 9x + 20}{x^2 - 4x}$.

8. Simplify $\frac{y^2 + 3y - 10}{y^2 - 25}$.

9. Simplify $\frac{m^2 - m - 12}{m^2 - 16}$.

Build Tasks – Answers



1. $\frac{x+3}{x}$

4. $\frac{x+3}{x}$

7. $\frac{x-5}{x}$

2. $\frac{x+4}{x}$

5. $\frac{x+2}{x}$

8. $\frac{y-2}{y+5}$

3. $\frac{x-5}{x}$

6. $\frac{x-3}{x}$

9. $\frac{m+3}{m+4}$

Build Tasks



10. Simplify $\frac{2x^2 + 10x}{4x}$.

11. Simplify $\frac{3y^2 - 12y}{6y}$.

12. Simplify $\frac{4a^2 + 12a}{8a}$.

13. Simplify $\frac{2x^2 - 18}{2x^2 + 6x}$.

14. Simplify $\frac{3x^2 - 27}{3x^2 - 3x}$.

15. Simplify $\frac{4x^2 - 20x + 24}{4x^2 - 8x}$.

16. Simplify $\frac{6a^2 + 13a - 5}{6a^2 - 2a}$.

17. Simplify $\frac{8m^2 - 22m + 15}{4m^2 - 10m}$.

18. Simplify $\frac{20x^2 - 13x - 21}{5x^2 + 15x}$.

Build Tasks – Answers



10. $\frac{x+5}{2}$

13. $\frac{x-3}{x}$

16. $\frac{2a+5}{2a}$

11. $\frac{y-4}{2}$

14. $\frac{(x-3)(x+3)}{x(x-1)}$

17. $\frac{(4m-5)(2m-3)}{2m(2m-5)}$

12. $\frac{a+3}{2}$

15. $\frac{x-3}{x}$

18. $\frac{(4x+3)(5x-7)}{5x(x+3)}$

Build Tasks



19. Simplify $\frac{x^2 - 9}{x^2 - 3x}$ and state restrictions.

20. Simplify $\frac{x^2 + 5x + 6}{x^2 + 2x}$ and state restrictions.

21. Simplify $\frac{x^2 - 5x + 6}{x^2 - 2x}$ and state restrictions.

22. A student cancels the x^2 in $\frac{x^2 - 9}{x^2 - 3x}$. Explain why this is wrong.

23. Which factor cancels in $\frac{y^2 + 3y - 10}{y^2 - 25}$?

24. Simplify $\frac{p^2 + 8p + 15}{p^2 + 3p}$.

25. Simplify $\frac{q^2 - 7q + 12}{q^2 - 4q}$.

26. Simplify $\frac{r^2 - r - 6}{r^2 + 2r}$.

27. Simplify $\frac{b^2 - 9}{b^2 + 6b + 9}$.

Build Tasks – Answers



19. $\frac{x+3}{x}, x \neq 0, 3$

22. You can only cancel factors after factorising, not separate terms.

25. $\frac{q-3}{q}$

20. $\frac{x+3}{x}, x \neq 0, -2$

23. $y+5$

26. $\frac{r-3}{r}$

21. $\frac{x-3}{x}, x \neq 0, 2$

24. $\frac{p+5}{p}$

27. $\frac{b-3}{b+3}$

Push Tasks



1. Simplify

$$\frac{6x^2 + 17x + 12}{3x^2 + 13x + 4}.$$

2. Simplify

$$\frac{8x^2 + 2x - 3}{4x^2 + 4x - 15}.$$

3. Simplify

$$\frac{12x^2 - 7x - 10}{10x^2 - 13x + 3}.$$

4. Simplify

$$\frac{15x^2 + 14x - 8}{5x^2 - 13x + 6}.$$

5. Simplify

$$\frac{14x^2 - 9x - 5}{4x^2 + 4x - 15}.$$

6. Simplify

$$\frac{12x^2 + 11x - 5}{20x^2 - 13x - 21}.$$

7. Simplify

$$\frac{18x^2 - 33x + 15}{12x^2 + 2x - 24}.$$

8. Simplify

$$\frac{16x^2 - 24x + 9}{28x^2 - 31x + 6}.$$

9. Simplify

$$\frac{45x^2 + 72x - 5}{30x^2 - 29x + 6}.$$

Push Tasks – Answers



1. $\frac{2x + 3}{x + 4}$

4. $\frac{3x + 4}{x - 2}$

7. $\frac{3(2x - 1)(3x - 5)}{2(3x - 4)(2x + 3)}$

2. $\frac{2x - 1}{2x + 5}$

5. $\frac{14x + 5}{(2x + 5)(x - 1)}$

8. $\frac{4x - 3}{7x - 6}$

3. $\frac{(3x + 2)(4x - 5)}{(10x - 3)(x - 1)}$

6. $\frac{3x - 1}{x + 3}$

9. $\frac{3x + 5}{3x - 2}$

Push Tasks



10. Simplify

$$\frac{x^2 - 4}{x^2 - 2x} \times \frac{x}{x + 2}. \text{ State restrictions.}$$

11. Simplify

$$\frac{x^2 + 5x + 6}{x^2 + 2x} \times \frac{x}{x + 3}. \text{ State restrictions.}$$

12. Simplify

$$\frac{x^2 - 5x + 6}{x^2 - 2x} \times \frac{x}{x - 3}. \text{ State restrictions.}$$

13. Simplify

$$\frac{x^2 - 9}{x^2 - 3x} \div \frac{x + 3}{x}.$$

14. Simplify

$$\frac{x^2 - 16}{x^2 - 4x} \div \frac{x + 4}{x}.$$

15. Simplify

$$\frac{x^2 - 25}{x^2 + 5x} \div \frac{x - 5}{x}.$$

16. Simplify

$$\frac{x^2 - 4}{x^2 - 2x} + \frac{1}{x}.$$

17. Simplify

$$\frac{x^2 - 9}{x^2 - 3x} - \frac{1}{x}.$$

18. Simplify

$$\frac{x^2 + 5x + 6}{x^2 + 2x} - \frac{1}{x}.$$

Push Tasks – Answers



10. $1, x \neq 0, 2, -2$

13. 1

16. $\frac{x+3}{x}$

11. $1, x \neq 0, -2, -3$

14. 1

17. $\frac{x+2}{x}$

12. $1, x \neq 0, 2, 3$

15. 1

18. $\frac{x+2}{x}$

Push Tasks



19. A student writes $\frac{x^2 - 9}{x^2 - 3x} = \frac{x - 3}{x}$. Explain the error.

20. A student writes $\frac{x^2 + 5x + 6}{x^2 + 2x} = \frac{x + 3}{x}$. Is this correct? Explain.

21. Factorise $(x^2 - 4)$ and $(x^2 - 2x)$, then simplify $\frac{x^2 - 4}{x^2 - 2x}$.

22. Create a fraction that simplifies to $\frac{x+2}{x}$ after factorising a quadratic numerator.

23. Create a fraction that simplifies to $\frac{x-3}{x+2}$ after factorising both numerator and denominator.

24. Which is easier to simplify: $\frac{x^2 - 9}{x^2 - 3x}$ or $\frac{45x^2 + 72x - 5}{30x^2 - 29x + 6}$? Give one reason.

25. Simplify $\frac{x^2 - 7x + 12}{x^2 - 5x + 6}$.

26. Simplify $\frac{x^2 + x - 6}{x^2 - 4x + 3}$.

27. Simplify $\frac{x^2 - 4x + 3}{x^2 - x - 6}$.

Push Tasks – Answers



19. They cancelled terms instead of factors. Correct:

$$\frac{(x-3)(x+3)}{x(x-3)} = \frac{x+3}{x}.$$

20. Yes.

$$\frac{(x+2)(x+3)}{x(x+2)} = \frac{x+3}{x},$$

with $x \neq 0, -2$.

21. $\frac{x+2}{x}$, with $x \neq 0, 2$

22. Example: $\frac{x^2 + 2x}{x}$

23. Example:

$$\frac{x^2 - 5x + 6}{x^2 - 4}$$

24. $\frac{x^2 - 9}{x^2 - 3x}$, because it is a direct difference-of-squares plus common factor

25. $\frac{x-4}{x-2}$

26. $\frac{(x+3)(x-2)}{(x-1)(x-3)}$

27. $\frac{x-1}{x+2}$