Probability assumptions

Mana Maths

Te reo Māori terms

tūponotanga
probability
Open in Te Aka

putanga
outcome
Open in Te Aka

ōrite
fair
Open in Te Aka

haukume
biased
Open in Te Aka

Probability assumptions

Probability Assumptions

Key idea

Counting outcomes assumes the device is **fair**, outcomes are **equally likely**, and the trial is **random**. If one fails, the method can fail too.

When it works

Fair die: $P(6) = \frac{1}{6}$ – 6 equally likely outcomes, 1 favourable.

When it fails

A spinner has 4 sectors, but one is twice as large. Outcomes are not equally likely.

Common mistake

A short run need not look balanced – a fair coin can still give 7 heads in 10 flips.

= copy this into your book. Boxes without a pencil are just to read.

Probability assumptions

Sample Size and Independence

Key idea

A short run varies a lot by chance – weak evidence. **Replacement** is a hidden assumption: it decides if the chance changes after each draw.

Sample size

6 in 10 flips is not shocking. 90 in 100 would be strong evidence of bias.

Replacement matters

3 red, 2 blue counters. **With replacement**, $P(\text{red}) = \frac{3}{5}$ every draw. **Without**, it changes.

Common mistake

Confusing "possible" with "likely". A fair device can still give an unusual result.

= **copy this into your book.** Boxes without a pencil are just to read.

Start Tasks

Identify fair chance devices and the assumptions behind equal probability.

1. A fair coin is tossed. Is $P(H) = P(T)$?

2. A fair die is rolled. Are all six outcomes equally likely?

3. What does “fair” mean in probability?

4. If a coin is fair, what is $P(H)$?

Start Tasks – Answers

Answers

1. Yes

2. Yes

3. All outcomes have equal chance

4. $\frac{1}{2}$

Start Tasks

Recognise when the “fair” assumption fails and explain the consequences.

5. A spinner has sections of different sizes. Can you still assume each colour is equally likely?

6. A coin lands heads 6 times in 10 tosses. Does that prove it is unfair?

7. Why is “random” important in a probability experiment?

8. Give one example of a biased chance device.

Start Tasks – Answers

Answers

5. No

6. No

7. It stops deliberate control

8. Example: weighted die

Build Tasks

Explain why the "equal outcomes" assumption matters when using the probability formula.

1. Why is $P(\text{head}) = \frac{1}{2}$ only valid if the coin is assumed fair?

2. A die has faces 1 to 6, but one side is heavier. Can we still use $\frac{1}{6}$ for each face? Explain.

3. A bag has 4 red and 6 blue counters. What assumption lets us say $P(\text{red}) = \frac{4}{10}$?

4. When is it reasonable to treat outcomes as equally likely?

Build Tasks – Answers

Answers

1. **Because $\frac{1}{2}$ assumes the coin is fair and outcomes are equally likely**

2. **No; if the die is weighted, outcomes are not equally likely**

3. **That each counter has an equal chance of being drawn (fair/random selection)**

4. **When the device is fair, balanced, and the experiment is random**

Build Tasks

Distinguish between what is possible and what is likely. Use sample size to judge evidence of bias.

5. A student gets 8 sixes in 20 rolls. Does that prove the die is biased? Why or why not?

6. Why does replacement matter when assuming the same probability on every draw?

7. Would 51 heads in 100 flips be strong evidence of bias? Explain briefly.

8. Why should large samples usually match theoretical probability more closely than small samples?

Build Tasks – Answers

Answers

5. No; 8 in 20 is 40%, close to 50%, so a fair die could produce this by chance

6. With replacement, the conditions stay the same each draw; without replacement, drawing changes the bag contents

7. No; 51 in 100 is 51%, very close to 50%, well within normal variation

8. Large samples average out random variation; small samples can vary widely by chance

Build Tasks

Identify hidden assumptions in probability statements and apply them to new situations.

9. If outcomes are not equally likely, what extra information do you need before finding a probability?

10. A game advertises a 1 in 4 chance of winning. What must be true for that claim to make sense?

11. Give a reason why a coin might behave unfairly in real life.

12. Describe one simple test you could do to check whether a spinner seems biased.

Build Tasks – Answers

Answers

9. The actual probability or frequency of each outcome

10. That the device is fair, outcomes are equally likely, and the process is truly random

11. Wear, scratches, or slight imbalance that favour one side

12. Spin it many times, record results, and check if frequencies are roughly equal

Push Tasks

Critique probability claims and reasoning. Explain why assumptions matter and when they fail.

1. A student says "The probability of head is $\frac{1}{2}$, so 10 heads in 10 flips is impossible." What is wrong with this claim?

2. A coin lands heads 18 times in 20 flips. Give one reason this does not automatically prove bias.

3. Give one reason 180 heads in 200 flips would be much more suspicious.

4. Why is a fair game not always a game where both players win equally often in a short run?

Push Tasks – Answers

Answers

1. Probability $\frac{1}{2}$ allows 10 heads in 10 flips; it's unlikely but not impossible

2. Small samples naturally vary; 18/20 = 90%, but a fair coin can produce this by chance

3. 180/200 = 90%, which is far from 50%; such a large sample producing such extreme results is much more suspic ious

4. Fairness means equal probability in the long run, not equal wins in short runs; random variation happens

Push Tasks

Analyse how violations of fairness assumptions affect probability calculations and experiment design.

5. A six-sided die is not a perfect cube. What assumption behind $\frac{1}{6}$ may fail?

6. A bag draw is without replacement. Why can the second-draw probability be different from the first?

7. Why does good experimental design matter when judging whether a device is biased?

8. A player chooses a card after peeking at marked backs. Is the experiment still random? Explain.

Push Tasks – Answers

Answers

5. **The assumption that each face has equal probability; if the cube is imperfect, the weight may not be even**

6. **The contents of the bag change after each draw; drawing one item removes it, so probabilities shift**

7. **Experimental bias (unequal spinning force, starting position, etc.) can favour some outcomes over others**

8. **No; the outcome is not random if the player has information or control over selection**

Push Tasks

Evaluate real-world probability scenarios and identify unstated assumptions or design flaws.

9. A casino says a wheel is fair because it has 20 labelled spaces. What extra assumption is missing?

10. Why might experimental probability be a better guide than theoretical probability for a biased device?

11. A student says "It hasn't rained for five days, so rain is now more likely." What mistake are they making?

12. Describe one situation where theoretical probability is a poor model for a real-world outcome.

Push Tasks – Answers

Answers

9. That each labelled space has equal chance of landing; labelling alone does not guarantee equal probability

10. A biased device shows its true probabilities; theoretical formula assumes fairness, so experimental data is more accurate

11. Gambler's fallacy; rainfall is not independent like fair coin flips; past events do not change future probabilities

12. Example: a bent or weighted die behaves differently than a fair die; theoretical model assumes fairness but reality breaks that assumption