



Multi-step Pythagoras' theory

Mana Maths

Te reo Māori terms



ture a Pythagoras

Pythagoras' theorem

Open in Te Aka

tapatoru hāngai

right-angle triangle

Open in Te Aka

hauroki

diagonal

Open in Te Aka

teitei

height

Open in Te Aka

Multi-Step Pythagoras



Key idea

Use Pythagoras to find a hidden length. Then use it in a **second** step: area, perimeter, or comparison.

Method

1. Find the hidden right triangle.
2. Get the missing side.
3. Use it in step two; add units.

Example: height then area

Isosceles: sides 13, base 10.
Half base = 5, so $h^2 = 13^2 - 5^2 = 144$, $h = 12$.
Area = $\frac{1}{2} \times 10 \times 12 = 60 \text{ cm}^2$.

Common mistake

Do not stop at the height or diagonal — finish the real target.

 = **copy this into your book.** Boxes without a pencil are just to read.

Two Steps in Action



Diagonal then perimeter

Rectangle 6×8 .

$$d^2 = 6^2 + 8^2 = 100, \text{ so } d = 10.$$

$$\text{Perimeter} = 2(6 + 8) = 28 \text{ cm.}$$

Common mistake

Write the intermediate length down clearly, so you do not mix it up with the final target.

Compare two routes

Rectangle 8×15 .

$$d^2 = 8^2 + 15^2 = 289, \text{ so } d = 17.$$

Two sides = 23, so the diagonal is 6 m shorter.

 = **copy this into your book.** Boxes without a pencil are just to read.

Exact Answers (Surds)



Key idea

Sometimes the length is not whole. Leave it **exact** as a surd unless asked to round.

Square from its diagonal

Square, diagonal 18, side s : $s^2 + s^2 = 18^2$.

$2s^2 = 324$, $s^2 = 162$, so $s = 9\sqrt{2}$ cm.

Common mistake

Do not round the intermediate length too early. Keep it exact, then round only the final answer.

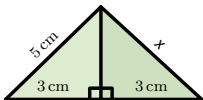
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Start Tasks

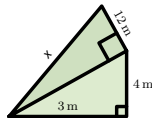


Find x . Find the hidden dimension first.

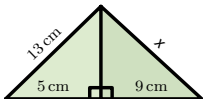
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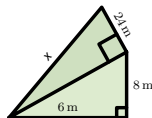
2.



3.



4.



Start Tasks — Answers



Find x . Find the hidden dimension first.

1.



Solution

$$h = \sqrt{10^2 - 6^2} = 8$$

$$x = \sqrt{8^2 + 15^2} =$$

$$17 \text{ cm}$$

2.



Solution

$$d = \sqrt{3^2 + 4^2} = 5$$

$$x = \sqrt{5^2 + 12^2} =$$

$$13 \text{ m}$$

3.



Solution

$$h = \sqrt{13^2 - 5^2} = 12$$

$$x = \sqrt{12^2 + 9^2} =$$

$$15 \text{ cm}$$

4.



Solution

$$d = \sqrt{6^2 + 8^2} = 10$$

$$x = \sqrt{10^2 + 24^2} =$$

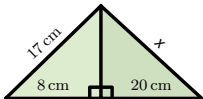
$$26 \text{ m}$$

Start Tasks

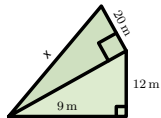


Find x . Show the two Pythagoras steps.

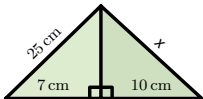
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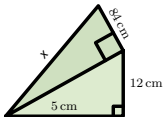
6.



7.



8.



Start Tasks — Answers



Find x . Show the two Pythagoras steps.

5.



Solution

$$h = \sqrt{17^2 - 8^2} = 15$$

$$x = \sqrt{15^2 + 20^2} = 25 \text{ cm}$$

6.



Solution

$$d = \sqrt{9^2 + 12^2} = 15$$

$$x = \sqrt{15^2 + 20^2} = 25 \text{ m}$$

7.



Solution

$$h = \sqrt{25^2 - 7^2} = 24$$

$$x = \sqrt{24^2 + 10^2} = 26 \text{ cm}$$

8.



Solution

$$d = \sqrt{5^2 + 12^2} = 13$$

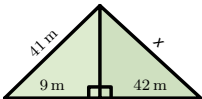
$$x = \sqrt{13^2 + 84^2} = 85 \text{ cm}$$

Start Tasks



Find x . Show both equations or steps.

9.



10. A rectangle is 9 cm by 12 cm. Find the diagonal, then the perimeter.

11. A square has area 64 m^2 . Find the side, then the diagonal.

12. An isosceles triangle has equal sides 17 cm and base 16 cm. Find the height, then the area.

Start Tasks — Answers



Find x . Show both equations or steps.

9.



Solution

$$h = \sqrt{41^2 - 9^2} = 40$$

$$x = \sqrt{40^2 + 42^2} = 58 \text{ m}$$

10. A rectangle is 9 cm by 12 cm. Find the diagonal, then the perimeter.

Solution

$$\text{Diagonal} = \sqrt{9^2 + 12^2} = 15$$

$$\text{Perimeter} = 2(9 + 12) = 42 \text{ cm}$$

11. A square has area 64 m^2 . Find the side, then the diagonal.

Solution

$$\text{Side} = \sqrt{64} = 8$$

$$\text{Diagonal} = 8\sqrt{2} \approx 11.3 \text{ m}$$

12. An isosceles triangle has equal sides 17 cm and base 16 cm. Find the height, then the area.

Solution

$$\text{Height} = \sqrt{17^2 - 8^2} = 15$$

$$\text{Area} = \frac{1}{2}(16)(15) = 120 \text{ cm}^2$$

Start Tasks



Find the hidden length first, then complete the second step.

13. A rectangle has diagonal 13 cm and width 5 cm. Find the length, then the perimeter.

14. A ladder reaches 8 m up a wall and the base is 6 m away. Find the ladder length, then round to 1 decimal place.

15. A square has side 7 m. Find the diagonal, then explain if a 10 m path is long enough.

16. An isosceles triangle has equal sides 13 cm and base 24 cm. Find the height, then the area.

Start Tasks — Answers



Find the hidden length first, then complete the second step.

13. A rectangle has diagonal 13 cm and width 5 cm. Find the length, then the perimeter.

Solution

$$\text{Length} = \sqrt{13^2 - 5^2} = 12$$

$$\text{Perimeter} = 2(5 + 12) = 34 \text{ cm}$$

15. A square has side 7 m. Find the diagonal, then explain if a 10 m path is long enough.

Solution

$$\text{Diagonal} = 7\sqrt{2} \approx 9.9 \text{ m}$$

Yes: $9.9 < 10$, long enough

14. A ladder reaches 8 m up a wall and the base is 6 m away. Find the ladder length, then round to 1 decimal place.

Solution

$$\begin{aligned}\text{Ladder} &= \sqrt{8^2 + 6^2} = 10 \\ &= 10.0 \text{ m (1 d.p.)}\end{aligned}$$

16. An isosceles triangle has equal sides 13 cm and base 24 cm. Find the height, then the area.

Solution

$$\text{Height} = \sqrt{13^2 - 12^2} = 5$$

$$\text{Area} = \frac{1}{2}(24)(5) = 60 \text{ cm}^2$$

Start Tasks



Two-step problems: find a hidden length first.

17. A rectangle is 3 m by 4 m. Find the diagonal, then the area.

18. A right triangle has hypotenuse 15 cm and one short side 9 cm. Find the other short side, then the area.

19. A square has diagonal 14 cm. Find the side, then the area.

20. A rectangle is 8 m by 15 m. Find the diagonal, then compare it with 23 m.

Start Tasks — Answers



Two-step problems: find a hidden length first.

17. A rectangle is 3 m by 4 m. Find the diagonal, then the area.

Solution

$$\text{Diagonal} = \sqrt{3^2 + 4^2} = 5$$

$$\text{Area} = 3 \times 4 = 12 \text{ m}^2$$

18. A right triangle has hypotenuse 15 cm and one short side 9 cm. Find the other short side, then the area.

Solution

$$\text{Other side} = \sqrt{15^2 - 9^2} = 12$$

$$\text{Area} = \frac{1}{2}(9)(12) = 54 \text{ cm}^2$$

19. A square has diagonal 14 cm. Find the side, then the area.

Solution

$$\text{Side} = \frac{14}{\sqrt{2}} = 7\sqrt{2} \approx 9.9$$

$$\text{Area} = \frac{14^2}{2} = 98 \text{ cm}^2$$

20. A rectangle is 8 m by 15 m. Find the diagonal, then compare it with 23 m.

Solution

$$\text{Diagonal} = \sqrt{8^2 + 15^2} = 17$$

$17 < 23$, so shorter than 23 m

Start Tasks



Find the missing dimension, then use it in your calculation.

21. An isosceles triangle has equal sides 25 cm and base 14 cm. Find the height, then the area.

22. A rectangle has diagonal 17 cm and width 8 cm. Find the length, then the perimeter.

23. A ladder is 25 m long and reaches 24 m high. Find the base distance, then the triangle area.

24. A square has side 9 cm. Find the diagonal exactly, then to 1 decimal place.

Start Tasks — Answers



Find the missing dimension, then use it in your calculation.

21. An isosceles triangle has equal sides 25 cm and base 14 cm. Find the height, then the area.

Solution

$$\text{Height} = \sqrt{25^2 - 7^2} = 24$$

$$\text{Area} = \frac{1}{2}(14)(24) = 168 \text{ cm}^2$$

22. A rectangle has diagonal 17 cm and width 8 cm. Find the length, then the perimeter.

Solution

$$\text{Length} = \sqrt{17^2 - 8^2} = 15$$

$$\text{Perimeter} = 2(8 + 15) = 46 \text{ cm}$$

23. A ladder is 25 m long and reaches 24 m high. Find the base distance, then the triangle area.

Solution

$$\text{Base} = \sqrt{25^2 - 24^2} = 7$$

$$\text{Area} = \frac{1}{2}(7)(24) = 84 \text{ m}^2$$

24. A square has side 9 cm. Find the diagonal exactly, then to 1 decimal place.

Solution

$$\text{Diagonal} = 9\sqrt{2} \text{ cm}$$

$$\approx 12.7 \text{ cm (1 d.p.)}$$

Start Tasks



Complete both steps: the hidden length, then the final target.

25. A triangle has area 24 cm^2 and base 8 cm . Find the height, then say if equal sides of 5 cm would fit an isosceles triangle.

26. A rectangle is 4 cm by 5 cm . Find the diagonal, then the perimeter.

27. A right triangle has hypotenuse 13 cm and one short side 12 cm . Find the other short side, then the area.

Start Tasks — Answers



Complete both steps: the hidden length, then the final target.

25. A triangle has area 24 cm^2 and base 8 cm . Find the height, then say if equal sides of 5 cm would fit an isosceles triangle.

Solution

$$\text{Height} = \frac{2(24)}{8} = 6$$

No: side needs $\sqrt{4^2 + 6^2} \approx 7.2 > 5$

27. A right triangle has hypotenuse 13 cm and one short side 12 cm . Find the other short side, then the area.

Solution

$$\text{Other side} = \sqrt{13^2 - 12^2} = 5$$

$$\text{Area} = \frac{1}{2}(12)(5) = 30 \text{ cm}^2$$

26. A rectangle is 4 cm by 5 cm . Find the diagonal, then the perimeter.

Solution

$$\text{Diagonal} = \sqrt{4^2 + 5^2} = \sqrt{41} \approx 6.4$$

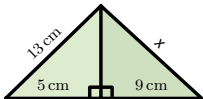
$$\text{Perimeter} = 2(4 + 5) = 18 \text{ cm}$$

Build Tasks

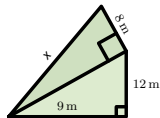


Find x . Show all steps clearly.

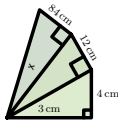
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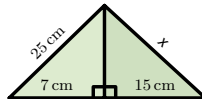
2.



3.



4.



Build Tasks — Answers



Find x . Show all steps clearly.

1.



Solution

$$h = \sqrt{13^2 - 5^2} = 12$$

$$x = \sqrt{12^2 + 9^2} =$$

$$15 \text{ cm}$$

2.



Solution

$$d = \sqrt{9^2 + 12^2} = 15$$

$$x = \sqrt{15^2 + 8^2} =$$

$$17 \text{ m}$$

3.



Solution

$$d_1 = \sqrt{3^2 + 4^2} = 5$$

$$d_2 = \sqrt{5^2 + 12^2} = 13$$

$$x = \sqrt{13^2 + 84^2} =$$

$$85 \text{ cm}$$

4.



Solution

$$h = \sqrt{25^2 - 7^2} = 24$$

$$x = \sqrt{24^2 + 15^2} =$$

$$\sqrt{801}$$

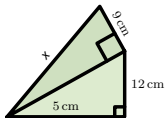
$$= 3\sqrt{89} \approx 28.3 \text{ cm}$$

Build Tasks

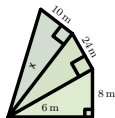


Find x . Label any hidden dimensions.

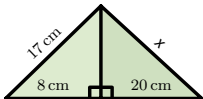
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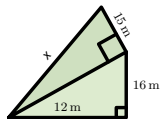
6.



7.



8.



Build Tasks — Answers



Find x . Label any hidden dimensions.

5.



Solution

$$d = \sqrt{5^2 + 12^2} = 13$$

$$x = \sqrt{13^2 + 9^2} =$$

$$\sqrt{250}$$

$$= 5\sqrt{10} \approx 15.8 \text{ cm}$$

6.



Solution

$$d_1 = \sqrt{6^2 + 8^2} = 10$$

$$d_2 = \sqrt{10^2 + 24^2} =$$

$$26$$

$$x = \sqrt{26^2 + 10^2} =$$

$$2\sqrt{194} \approx 27.9 \text{ m}$$

7.



Solution

$$h = \sqrt{17^2 - 8^2} = 15$$

$$x = \sqrt{15^2 + 20^2} =$$

$$25 \text{ cm}$$

8.



Solution

$$d = \sqrt{12^2 + 16^2} = 20$$

$$x = \sqrt{20^2 + 15^2} =$$

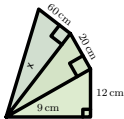
$$25 \text{ m}$$

Build Tasks



Find x . Do not round intermediate values.

9.



10. A rectangle is 7 cm by 24 cm. Find the diagonal, then the area and perimeter.

11. A square has diagonal 20 m. Find the side, then the area to 1 decimal place.

12. An isosceles triangle has equal sides 17 cm and base 30 cm. Find the height, then the area.

Build Tasks — Answers



Find x . Do not round intermediate values.

9.



Solution

$$d_1 = \sqrt{9^2 + 12^2} = 15$$

$$d_2 = \sqrt{15^2 + 20^2} = 25$$

$$x = \sqrt{25^2 + 60^2} = 65 \text{ cm}$$

10. A rectangle is 7 cm by 24 cm. Find the diagonal, then the area and perimeter.

Solution

$$\text{Diagonal} = \sqrt{7^2 + 24^2} = 25$$

$$\text{Area} = 168 \text{ cm}^2, \text{ Perimeter} = 62 \text{ cm}$$

11. A square has diagonal 20 m. Find the side, then the area to 1 decimal place.

Solution

$$\text{Side} = \frac{20}{\sqrt{2}} = 10\sqrt{2} \approx 14.1$$

$$\text{Area} = \frac{20^2}{2} = 200.0 \text{ m}^2$$

12. An isosceles triangle has equal sides 17 cm and base 30 cm. Find the height, then the area.

Solution

$$\text{Height} = \sqrt{17^2 - 15^2} = 8$$

$$\text{Area} = \frac{1}{2}(30)(8) = 120 \text{ cm}^2$$

Build Tasks



Find the intermediate value, then use it for the final step.

13. A ladder is 25 m long and reaches 24 m high. Find the base distance, then the triangle area.

14. A rectangle has perimeter 82 cm and diagonal 29 cm. Find its side lengths.

15. A square has side 9 cm. Find the diagonal exactly and to 1 decimal place, then the perimeter.

16. A triangle has area 40 cm^2 and base 10 cm. If it is isosceles, find the height, then each equal side.

Build Tasks — Answers



Find the intermediate value, then use it for the final step.

13. A ladder is 25 m long and reaches 24 m high. Find the base distance, then the triangle area.

Solution

$$\text{Base} = \sqrt{25^2 - 24^2} = 7$$

$$\text{Area} = \frac{1}{2}(7)(24) = 84 \text{ m}^2$$

15. A square has side 9 cm. Find the diagonal exactly and to 1 decimal place, then the perimeter.

Solution

$$\text{Diagonal} = 9\sqrt{2} \approx 12.7 \text{ cm}$$

$$\text{Perimeter} = 36 \text{ cm}$$

14. A rectangle has perimeter 82 cm and diagonal 29 cm. Find its side lengths.

Solution

$$l + w = 41, \text{ so } lw = \frac{41^2 - 29^2}{2} = 420$$

Sides 20 cm and 21 cm

16. A triangle has area 40 cm^2 and base 10 cm. If it is isosceles, find the height, then each equal side.

Solution

$$\text{Height} = \frac{2(40)}{10} = 8$$

$$\text{Each side} = \sqrt{5^2 + 8^2} = \sqrt{89} \approx 9.4 \text{ cm}$$

Build Tasks



Solve in two steps: find a hidden length first.

17. A rectangle is 11 cm by 60 cm. Find the diagonal, then decide if a 61 cm brace fits exactly.

18. A right triangle has one short side 20 cm and hypotenuse 29 cm. Find the other short side, then the perimeter.

19. Find the distance from (1, 1) to (13, 6), then give the decimal answer to 1 decimal place.

20. An equilateral triangle has side 10 mm. Find its height, then the area to 1 decimal place.

Build Tasks — Answers



Solve in two steps: find a hidden length first.

17. A rectangle is 11 cm by 60 cm. Find the diagonal, then decide if a 61 cm brace fits exactly.

Solution

$$\text{Diagonal} = \sqrt{11^2 + 60^2} = 61$$

Yes, the 61 cm brace fits exactly

19. Find the distance from (1, 1) to (13, 6), then give the decimal answer to 1 decimal place.

Solution

$$\begin{aligned}\text{Distance} &= \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \\ &= 13.0 \text{ (1 d.p.)}\end{aligned}$$

18. A right triangle has one short side 20 cm and hypotenuse 29 cm. Find the other short side, then the perimeter.

Solution

$$\text{Other side} = \sqrt{29^2 - 20^2} = 21$$

$$\text{Perimeter} = 20 + 21 + 29 = 70 \text{ cm}$$

20. An equilateral triangle has side 10 mm. Find its height, then the area to 1 decimal place.

Solution

$$\text{Height} = \sqrt{10^2 - 5^2} = 5\sqrt{3} \approx 8.7$$

$$\text{Area} = \frac{1}{2}(10)(5\sqrt{3}) = 25\sqrt{3} \approx 43.3 \text{ mm}^2$$

Build Tasks



Complete two steps: first find the hidden dimension.

21. A regular hexagon has side 8 cm. Find the height of one small triangle, then the total area.

22. An isosceles triangle has sides 6.5 m, 6.5 m, and 7.4 m. Find the height, then the area.

23. A rectangle is 7 cm longer than it is wide and its diagonal is 13 cm. Find the dimensions, then the area.

24. A boat sails 80 km east and then 110 km south. Find how far it is from the start, then round to 1 decimal place.

Build Tasks — Answers



Complete two steps: first find the hidden dimension.

21. A regular hexagon has side 8 cm. Find the height of one small triangle, then the total area.

Solution

Small-triangle height

$$= \sqrt{8^2 - 4^2} = 4\sqrt{3} \approx 6.93$$

$$\text{Total area} = 96\sqrt{3} \approx 166.3 \text{ cm}^2$$

23. A rectangle is 7 cm longer than it is wide and its diagonal is 13 cm. Find the dimensions, then the area.

Solution

$$w^2 + (w + 7)^2 = 169 \Rightarrow w = 5$$

$$5 \text{ cm by } 12 \text{ cm, Area} = 60 \text{ cm}^2$$

22. An isosceles triangle has sides 6.5 m, 6.5 m, and 7.4 m. Find the height, then the area.

Solution

$$\text{Height} = \sqrt{6.5^2 - 3.7^2} \approx 5.34$$

$$\text{Area} = \frac{1}{2}(7.4)(5.34) \approx 19.8 \text{ m}^2$$

24. A boat sails 80 km east and then 110 km south. Find how far it is from the start, then round to 1 decimal place.

Solution

$$\text{Distance} = \sqrt{80^2 + 110^2} = \sqrt{18500}$$

$$\approx 136.0 \text{ km}$$

Build Tasks



Use Pythagoras first, then complete the second step.

25. A football pitch is 105 m by 68 m. Find the diagonal, then compare it with 125 m.

26. A rectangle has width 12 cm and perimeter 58 cm. Find the diagonal.

27. Explain why a multi-step Pythagoras problem often starts by finding a hidden height or diagonal first.

Build Tasks — Answers



Use Pythagoras first, then complete the second step.

25. A football pitch is 105 m by 68 m. Find the diagonal, then compare it with 125 m.

Solution

$$\begin{aligned}\text{Diagonal} &= \sqrt{105^2 + 68^2} = \sqrt{15649} \\ &\approx 125.1 \text{ m, just over 125 m}\end{aligned}$$

27. Explain why a multi-step Pythagoras problem often starts by finding a hidden height or diagonal first.

Solution

It is a side of the right triangle you need, so find it before the final step.

26. A rectangle has width 12 cm and perimeter 58 cm. Find the diagonal.

Solution

$$\text{Length} = 29 - 12 = 17$$

Diagonal

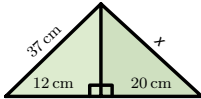
$$= \sqrt{12^2 + 17^2} = \sqrt{433} \approx 20.8 \text{ cm}$$

Push Tasks

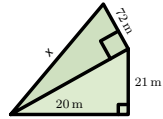


Find x exactly and to 1 d.p. Show all steps.

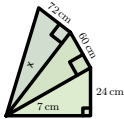
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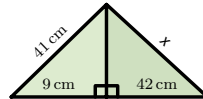
2.



3.



4.



Push Tasks — Answers



Find x exactly and to 1 d.p. Show all steps.

1.



Solution

$$\begin{aligned}h &= \sqrt{37^2 - 12^2} = 35 \\x &= \sqrt{35^2 + 20^2} = \sqrt{1625} \\&= 5\sqrt{65} \approx 40.3 \text{ cm}\end{aligned}$$

2.



Solution

$$\begin{aligned}d &= \sqrt{20^2 + 21^2} = 29 \\x &= \sqrt{29^2 + 72^2} = \sqrt{6025} \\&= 5\sqrt{241} \approx 77.6 \text{ m}\end{aligned}$$

3.



Solution

$$\begin{aligned}d_1 &= \sqrt{7^2 + 24^2} = 25 \\d_2 &= \sqrt{25^2 + 60^2} = 65 \\x &= \sqrt{65^2 + 72^2} = 97 \text{ cm}\end{aligned}$$

4.



Solution

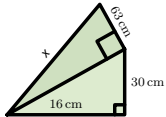
$$\begin{aligned}h &= \sqrt{41^2 - 9^2} = 40 \\x &= \sqrt{40^2 + 42^2} = 58 \text{ cm}\end{aligned}$$

Push Tasks

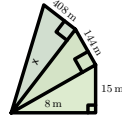


Find x . Keep intermediate values exact.

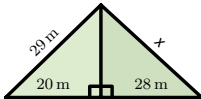
5.



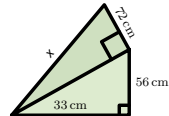
6.



7.



8.



Push Tasks — Answers



Find x . Keep intermediate values exact.

5.



Solution

$$d = \sqrt{16^2 + 30^2} = 34$$

$$x = \sqrt{34^2 + 63^2} =$$

$$\sqrt{5125}$$

$$= 5\sqrt{205} \approx 71.6 \text{ cm}$$

6.



Solution

$$d_1 = \sqrt{8^2 + 15^2} = 17$$

$$d_2 = \sqrt{17^2 + 144^2} =$$

$$145$$

$$x = \sqrt{145^2 + 408^2} =$$

$$433 \text{ m}$$

7.



Solution

$$h = \sqrt{29^2 - 20^2} = 21$$

$$x = \sqrt{21^2 + 28^2} =$$

$$35 \text{ m}$$

8.



Solution

$$d = \sqrt{33^2 + 56^2} = 65$$

$$x = \sqrt{65^2 + 72^2} =$$

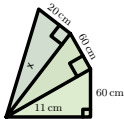
$$97 \text{ cm}$$

Push Tasks



Find x . Do not round along the way.

9.



10. An isosceles triangle has equal sides 29 cm and height 21 cm. Find the base, then the area.

11. A square has area 200 m^2 . Find the side, then the diagonal to 1 decimal place.

12. A rectangle is 24 m by 7 m. Compare the diagonal route with walking along two sides.

Push Tasks — Answers



Find x . Do not round along the way.

9.



Solution

$$d_1 = \sqrt{11^2 + 60^2} = 61$$

$$d_2 = \sqrt{61^2 + 60^2} = \sqrt{7321}$$

$$x = \sqrt{7321 + 20^2} = \sqrt{7721} \approx 87.9 \text{ cm}$$

11. A square has area 200 m^2 . Find the side, then the diagonal to 1 decimal place.

Solution

$$\text{Side} = \sqrt{200} = 10\sqrt{2} \approx 14.1$$

$$\text{Diagonal} = \sqrt{2} \cdot 10\sqrt{2} = 20.0 \text{ m}$$

10. An isosceles triangle has equal sides 29 cm and height 21 cm. Find the base, then the area.

Solution

$$\text{Half base} = \sqrt{29^2 - 21^2} = 20, \text{ base} = 40$$

$$\text{Area} = \frac{1}{2}(40)(21) = 420 \text{ cm}^2$$

12. A rectangle is 24 m by 7 m. Compare the diagonal route with walking along two sides.

Solution

$$\text{Diagonal} = \sqrt{24^2 + 7^2} = 25$$

$$\text{Two sides} = 31; \text{ diagonal saves } 6 \text{ m}$$

Push Tasks



Solve multi-step problems: find intermediate values exactly.

13. An isosceles triangle has sides 6.5 m, 6.5 m, and 7.4 m. Find the height, then the area, then the perimeter.

14. A rectangle has diagonal 20 cm and length 4 cm longer than its width. Find the dimensions.

15. A right triangle has hypotenuse 65 cm and one short side 16 cm. Find the other short side, then the area.

16. A boat sails 80 km east and 110 km south. Find the direct distance, then round to 1 decimal place and compare with 136 km.

Push Tasks — Answers



Solve multi-step problems: find intermediate values exactly.

13. An isosceles triangle has sides 6.5 m, 6.5 m, and 7.4 m. Find the height, then the area, then the perimeter.

Solution

$$\text{Height} = \sqrt{6.5^2 - 3.7^2} \approx 5.34$$

$$\text{Area} \approx 19.8 \text{ m}^2, \text{ Perimeter} = 20.4 \text{ m}$$

15. A right triangle has hypotenuse 65 cm and one short side 16 cm. Find the other short side, then the area.

Solution

$$\text{Other side} = \sqrt{65^2 - 16^2} = 63$$

$$\text{Area} = \frac{1}{2}(16)(63) = 504 \text{ cm}^2$$

14. A rectangle has diagonal 20 cm and length 4 cm longer than its width. Find the dimensions.

Solution

$$w^2 + (w + 4)^2 = 400 \Rightarrow w = 12$$

$$12 \text{ cm by } 16 \text{ cm}$$

16. A boat sails 80 km east and 110 km south. Find the direct distance, then round to 1 decimal place and compare with 136 km.

Solution

$$\text{Distance} = \sqrt{80^2 + 110^2} = \sqrt{18500}$$

$$\approx 136.0 \text{ km, about equal to } 136 \text{ km}$$

Push Tasks



Complete both steps: find the hidden length, then reason about the result.

17. A screen is 48 cm wide and 36 cm high. Find the diagonal, then explain why it is a scaled triple.

18. A square has diagonal 14 cm. Find the side to 1 decimal place, then the perimeter to 1 decimal place.

19. A triangle has area 40 cm^2 and base 10 cm. If it is isosceles, find each equal side, then the perimeter.

20. A rectangle is 11 cm by 60 cm. Find its diagonal exactly, then explain why 61 cm is special.

Push Tasks — Answers



Complete both steps: find the hidden length, then reason about the result.

17. A screen is 48 cm wide and 36 cm high. Find the diagonal, then explain why it is a scaled triple.

Solution

$$\begin{aligned}\text{Diagonal} &= \sqrt{48^2 + 36^2} = 60 \\ (36, 48, 60) &= 12 \times (3, 4, 5)\end{aligned}$$

18. A square has diagonal 14 cm. Find the side to 1 decimal place, then the perimeter to 1 decimal place.

Solution

$$\begin{aligned}\text{Side} &= \frac{14}{\sqrt{2}} = 7\sqrt{2} \approx 9.9 \\ \text{Perimeter} &= 28\sqrt{2} \approx 39.6 \text{ cm}\end{aligned}$$

19. A triangle has area 40 cm^2 and base 10 cm. If it is isosceles, find each equal side, then the perimeter.

Solution

$$\begin{aligned}\text{Each side} &= \sqrt{5^2 + 8^2} = \sqrt{89} \\ \text{Perimeter} &= 10 + 2\sqrt{89} \approx 28.9 \text{ cm}\end{aligned}$$

20. A rectangle is 11 cm by 60 cm. Find its diagonal exactly, then explain why 61 cm is special.

Solution

$$\begin{aligned}\text{Diagonal} &= \sqrt{11^2 + 60^2} = 61 \\ \text{Since } 11^2 + 60^2 &= 61^2 \text{ (a triple)}\end{aligned}$$

Push Tasks



Multi-step: use Pythagoras, then complete the calculation.

21. A regular hexagon has side 8 cm. Find its total area exactly in surd form or to 1 decimal place.

22. A rectangle has diagonal 13 cm and is 7 cm longer than it is wide. Find the width, length, and area.

23. The distance from $(1, 1)$ to $(13, 6)$ is used as the hypotenuse of another right triangle with one short side 5. Find the other short side exactly.

24. A square patio has side 7 m. Find the diagonal, then say if a 10 m cover strip is enough.

Push Tasks — Answers



Multi-step: use Pythagoras, then complete the calculation.

21. A regular hexagon has side 8 cm. Find its total area exactly in surd form or to 1 decimal place.

Solution

Six equilateral triangles, height = $4\sqrt{3}$
Area = $96\sqrt{3} \approx 166.3 \text{ cm}^2$

23. The distance from (1, 1) to (13, 6) is used as the hypotenuse of another right triangle with one short side 5. Find the other short side exactly.

Solution

Distance = $\sqrt{12^2 + 5^2} = 13$
Other side = $\sqrt{13^2 - 5^2} = 12$

22. A rectangle has diagonal 13 cm and is 7 cm longer than it is wide. Find the width, length, and area.

Solution

$w^2 + (w + 7)^2 = 169 \Rightarrow w = 5$
5 cm by 12 cm, Area = 60 cm^2

24. A square patio has side 7 m. Find the diagonal, then say if a 10 m cover strip is enough.

Solution

Diagonal = $7\sqrt{2} \approx 9.9 \text{ m}$
Yes: $9.9 < 10$, the strip is enough

Push Tasks



Solve and explain: two steps including reasoning about your answer.

25. A right triangle has short sides 9 cm and 12 cm. Find the hypotenuse, then compare it with 15 cm.

26. Why is 'find the hidden height first' such a common move in multi-step Pythagoras questions?

27. Write your own two-step Pythagoras question where the first answer is used to find an area or perimeter.

Push Tasks — Answers



Solve and explain: two steps including reasoning about your answer.

25. A right triangle has short sides 9 cm and 12 cm. Find the hypotenuse, then compare it with 15 cm.

Solution

Hypotenuse = $\sqrt{9^2 + 12^2} = 15$
Exactly equal to 15 cm

27. Write your own two-step Pythagoras question where the first answer is used to find an area or perimeter.

Solution

Answers vary, e.g. a 6×8 rectangle:
diagonal = 10, then perimeter = 24 cm.

26. Why is 'find the hidden height first' such a common move in multi-step Pythagoras questions?

Solution

The final target needs a length not given at the start; Pythagoras supplies it for the second step.