



Limits

Mana Maths

Te reo Māori terms



tepe

limit

Open in Te Aka

whāngai ki

tends to

Open in Te Aka

tauwehea

factorise

Open in Te Aka

**pārōnaki o te
pātapa**

gradient of tangent

Open in Te Aka

Limits - Notes and Steps



Key idea

A **limit** asks what value $f(x)$ heads towards as x gets close to a .

Written $\lim_{x \rightarrow a} f(x)$.

First try: substitute

Put $x = a$ in. A number means that is the limit.

$$\lim_{x \rightarrow 2} (x + 3) = 5$$

Common mistake

Substitution fails when the bottom becomes 0, giving $\frac{0}{0}$.

$\frac{0}{0}$ is **not** 0 or 1 — it means "try another way" (next slides).



When You Get 0/0: Factorise



Key idea



If substitution gives $\frac{0}{0}$:
factorise top and bottom, **cancel**
the common factor, then substitute again.

Example



$$\begin{aligned}\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} \\&= \lim_{x \rightarrow 5} \frac{(x - 5)(x + 5)}{x - 5} \\&= \lim_{x \rightarrow 5} (x + 5) = 10\end{aligned}$$

Common mistake

Giving up at $\frac{0}{0}$.
Cancel the $(x - 5)$ first, then put
 $x = 5$.

Limits at Infinity



Key idea

For $x \rightarrow \infty$, divide every term by the **highest power** of x .

Then $\frac{1}{x}, \frac{1}{x^2} \rightarrow 0$.

Example

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{5x^2 - x + 1}$$

Divide by x^2 : $\frac{3 + \frac{2}{x}}{5 - \frac{1}{x} + \frac{1}{x^2}} \rightarrow \frac{3}{5}$

Common mistake

Compare the **leading** coefficients: the answer is $\frac{3}{5}$, not 3 or 5.

The h to 0 Form



Key idea

For $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$:

expand and **simplify first**, then let $h \rightarrow 0$.

Example

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ = \frac{2xh + h^2}{h} = 2x + h\end{aligned}$$

As $h \rightarrow 0$: $= 2x$

Common mistake

Putting $h = 0$ too early gives $\frac{0}{0}$.
Cancel the h first, then let $h \rightarrow 0$.



= copy this into your book. Boxes without a pencil are just to read.

Start Tasks



1. Evaluate: $\lim_{x \rightarrow 3} (x + 2)$

2. Evaluate: $\lim_{x \rightarrow 4} (2x - 1)$

3. Evaluate:
 $\lim_{x \rightarrow -1} (x^2 + 3)$

4. Evaluate: $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

5. Evaluate: $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

6. Evaluate: $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

7. Evaluate:
 $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2}$

8. Evaluate:
 $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$

9. Evaluate:
 $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

Start Tasks - Answers



1. 5

2. 7

3. 4

4. 4

5. 6

6. 2

7. -4

8. 8

9. 10

Start Tasks



10. Evaluate:

$$\lim_{x \rightarrow -3} \frac{x^2 + 3x}{x + 3}$$

11. Evaluate:

$$\lim_{x \rightarrow 0} \frac{x^2 + 2x}{x}$$

12. Evaluate:

$$\lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1}$$

13. Evaluate: $\lim_{x \rightarrow \infty} \frac{1}{x}$

14. Evaluate: $\lim_{x \rightarrow \infty} \frac{5}{x}$

15. Evaluate: $\lim_{x \rightarrow \infty} \frac{3}{x^2}$

16. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{2x + 1}{x + 3}$$

17. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{3x - 2}{x + 1}$$

18. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{x + 4}{2x - 1}$$

Start Tasks - Answers



10. -3

11. 2

12. 1

13. 0

14. 0

15. 0

16. 2

17. 3

18. $\frac{1}{2}$

Start Tasks



19. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h) - x}{h}$$

20. Evaluate: $\lim_{h \rightarrow 0} \frac{2h}{h}$

21. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h) + x}{h}$$

22. Evaluate:

$$\lim_{x \rightarrow 1} (x^2 + 1)$$

23. Evaluate:

$$\lim_{x \rightarrow 0} (x^2 + 2x + 1)$$

24. Evaluate:

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$$

25. Evaluate:

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$$

26. Evaluate:

$$\lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{x + 1}$$

27. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{4x - 3}{2x + 5}$$

Start Tasks - Answers



19. 1

20. 2

21. Does not exist
(undefined)

22. 2

23. 1

24. 12

25. 5

26. 0

27. 2

Build Tasks



1. Evaluate:

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$$

2. Evaluate:

$$\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^2 - 4}$$

3. Evaluate:

$$\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9}$$

4. Evaluate:

$$\lim_{x \rightarrow -2} \frac{x^2 + x - 2}{x^2 + 3x + 2}$$

5. Evaluate: $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$

6. Evaluate: $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$

7. Evaluate:

$$\lim_{x \rightarrow 6} \frac{x^2 - 5x - 6}{x^2 - 36}$$

8. Evaluate:

$$\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x - 3}$$

9. Evaluate:

$$\lim_{x \rightarrow -3} \frac{x^2 + 6x + 9}{x^2 - 9}$$

Build Tasks - Answers



1. $\frac{5}{4}$

2. $\frac{3}{2}$

3. $\frac{1}{6}$

4. -3

5. 3

6. 3

7. $\frac{7}{12}$

8. 7

9. 0

Build Tasks



10. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{5x^2 + 2}{3x^2 - 1}$$

11. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{2x^2 - x}{x^2 + 3x}$$

12. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 4x}{x^2 - 5}$$

13. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{x^2 + 2x + 1}{2x^2 - x + 3}$$

14. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{4x^2 - 3x + 2}{5x^2 + 4x - 1}$$

15. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{7x^3 + 2x}{3x^3 - x^2}$$

16. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2}{2x^3 + 5x}$$

17. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{2x^3 + 5}{4x^3 - 3x + 1}$$

18. Evaluate:

$$\lim_{x \rightarrow \infty} \frac{3x^3 + x^2 - x}{2x^3 + 4x^2 + 1}$$

Build Tasks - Answers



10. $\frac{5}{3}$

11. 2

12. 3

13. $\frac{1}{2}$

14. $\frac{4}{5}$

15. $\frac{7}{3}$

16. $\frac{1}{2}$

17. $\frac{1}{2}$

18. $\frac{3}{2}$

Build Tasks



19. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

20. Evaluate:

$$\lim_{h \rightarrow 0} \frac{2(x+h)^2 - 2x^2}{h}$$

21. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 + 5(x+h) - (x^2 + 5x)}{h}$$

22. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

23. Evaluate:

$$\lim_{h \rightarrow 0} \frac{3(x+h)^2 + 2(x+h) - (3x^2 + 2x)}{h}$$

24. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$

25. Evaluate:

$$\lim_{h \rightarrow 0} \frac{5(x+h)^2 - (x+h) - (5x^2 - x)}{h}$$

26. Evaluate:

$$\lim_{h \rightarrow 0} \frac{4(x+h)^2 - (4x^2)}{h}$$

27. Evaluate:

$$\lim_{h \rightarrow 0} \frac{-2(x+h)^2 + 6(x+h) - (-2x^2 + 6x)}{h}$$

Build Tasks - Answers



19. $2x$

20. $4x$

21. $2x + 5$

22. $2x - 3$

23. $6x + 2$

24. $2x + 2$

25. $10x - 1$

26. $8x$

27. $-4x + 6$

Push Tasks



1. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

2. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(a+h)^3 - a^3}{h}$$

3. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$$

4. Evaluate:

$$\lim_{h \rightarrow 0} \frac{2(x+h)^3 - 2x^3}{h}$$

5. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 + (x+h) - (x^3 + x)}{h}$$

6. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - 4(x+h) - (x^3 - 4x)}{h}$$

7. Evaluate:

$$\lim_{h \rightarrow 0} \frac{3(x+h)^3 - 2(x+h) - (3x^3 - 2x)}{h}$$

8. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(b+h)^4 - b^4}{h}$$

9. Evaluate:

$$\lim_{h \rightarrow 0} \frac{2(a+h)^3 + (a+h) - (2a^3 + a)}{h}$$

Push Tasks - Answers



1. $3x^2$

2. $3a^2$

3. $4x^3$

4. $6x^2$

5. $3x^2 + 1$

6. $3x^2 - 4$

7. $9x^2 - 2$

8. $4b^3$

9. $6a^2 + 1$

Push Tasks



10. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) + 1}{h}$$

11. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - (x+h) - (x^3 - x)}{h}$$

12. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 + 5 - (x^2 + 5)}{h}$$

13. Evaluate:

$$\lim_{h \rightarrow 0} \frac{4(x+h)^2 - 3(x+h) + 1}{h}$$

14. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 + 2(x+h)^2 - (x^3 + 2x^2)}{h}$$

15. Evaluate:

$$\lim_{h \rightarrow 0} \frac{2(x+h)^3 - 5(x+h) - (2x^3 - 5x)}{h}$$

16. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(a+h)^3 - (a+h)^2 - (a^3 - a^2)}{h}$$

17. Evaluate:

$$\lim_{h \rightarrow 0} \frac{5(x+h)^3 + 3(x+h)^2 - (5x^3 + 3x^2)}{h}$$

18. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^4 + (x+h)^2 - (x^4 + x^2)}{h}$$

Push Tasks - Answers



10. $2x + 2$

11. $3x^2 - 1$

12. $2x$

13. $8x - 3$

14. $3x^2 + 4x$

15. $6x^2 - 5$

16. $3a^2 - 2a$

17. $15x^2 + 6x$

18. $4x^3 + 2x$

Push Tasks



19. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 + (x+h)^2 + (x+h) - x^3 - x^2 - x}{h}$$

20. Evaluate:

$$\lim_{h \rightarrow 0} \frac{2(a+h)^3 - 3(a+h) - 2a^3 + 3a}{h}$$

21. Evaluate:

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 + 4x - (x^3 + 4x)}{h}$$

22. Explain what it means for a limit to exist at a point $x = a$, and give an example of a function where the

23. If

$$\lim_{x \rightarrow 2} \frac{x^2 + kx - 10}{x - 2} = 7,$$

find the value of k .

24. If

$$\lim_{x \rightarrow 3} \frac{x^2 + ax + b}{x - 3} = 5,$$

find the values of a and b .

25. Find

$$\lim_{x \rightarrow \infty} \frac{3x^4 + 2x}{2x^4 + 5x^2 - 1}$$

and explain your reasoning.

26. Find

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ for } f(x) = x^3 - 2x^2 + 3x - 1.$$

27. Show that

$$\lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} = nx^{n-1} \text{ for } n = 4,$$

explaining each step.

Push Tasks - Answers



19. $3x^2 + 2x + 1$

20. $6a^2 - 3$

21. $3x^2$

22. Limit exists if LHS limit = RHS limit. E.g.

$f(x) = \frac{1}{x-1}$ has no

limit at $x = 1$

23. $k = 3$

24. $a = -1, b = -6$

25. $\frac{3}{2}$ (highest powers rule)

26. $3x^2 - 4x + 3$

27. $\frac{(x+h)^4 - x^4}{h} = 4x^3 + 6x^2h + 4xh^2 + h^3$,
then $h \rightarrow 0$ gives $4x^3$