

Te reo Māori terms



kauwhata

graph

Open in Te Aka

tau whakarea

coefficient

Open in Te Aka

koi

vertex

Open in Te Aka

pito hurihuri

turning point

Open in Te Aka

Graphing parabolas 1

Notes & Steps



Key idea

A parabola in vertex form can be written as $y = a(x - h)^2 + k$. The vertex is (h, k) , the axis is $x = h$, and a controls the opening direction and how steep or wide the graph looks.

What you can read fast

1. If $a > 0$, the graph opens upward.
2. If $a < 0$, the graph opens downward.
3. If $|a| > 1$, the graph is narrower than $y = x^2$.
4. If $0 < |a| < 1$, the graph is wider than $y = x^2$.

Common mistake

Thinking a bigger a moves the vertex. It does not. Changing a changes the shape and direction, but the vertex stays at (h, k) .

Useful reminders

5. The vertex stays (h, k) even when $a \neq 1$.
6. Points the same distance from the axis still have the same y -value.
7. Use the vertex first, then one point on each side, then intercepts if needed.
8. A negative a reflects the graph across the x -axis.

Graphing parabolas 1

Notes & Steps

8

Example 1: reading the graph

For $y = 2(x - 1)^2 - 3$, the vertex is $(1, -3)$ and the axis is $x = 1$. Because $a = 2$, the graph opens upward and is narrower than $y = x^2$.

Example 2: using symmetry

Still using $y = 2(x - 1)^2 - 3$:

$$y(0) = 2(0-1)^2 - 3 = -1, \quad y(2) = 2(2-1)^2 - 3 = -1$$

The inputs 0 and 2 are both one unit from the axis, so they match.

Try these

1. State the vertex and axis of $y = -3(x + 2)^2 + 4$.
2. Is $y = \frac{1}{2}(x - 5)^2 + 1$ wider or narrower than $y = x^2$?
3. Find y when $x = 0$ for $y = 2(x - 3)^2 - 4$.

Graphing parabolas 1

Notes & Steps



Example 3: finding roots

For $y = 2(x - 1)^2 - 8$, set $y = 0$:

$$2(x - 1)^2 = 8 \Rightarrow (x - 1)^2 = 4$$

So $x - 1 = \pm 2$, giving roots $x = -1$ and $x = 3$.

Example 4: finding a

If the vertex is $(2, -1)$ and the graph passes through $(3, 1)$, then

$$1 = a(3 - 2)^2 - 1$$

so $1 = a - 1$, giving $a = 2$. The equation is $y = 2(x - 2)^2 - 1$.

Common mistake

Using only the vertex to sketch and forgetting to test another point. Different values of a can share the same vertex but have very different widths.

Start Tasks



1. For $y = 2(x - 1)^2 + 3$, write the vertex.

2. For $y = -3(x + 2)^2 + 1$, write the vertex.

3. For $y = \frac{1}{2}(x - 4)^2 - 5$, write the vertex.

4. For $y = 2(x - 1)^2 + 3$, what is the axis of symmetry?

5. For $y = -3(x + 2)^2 + 1$, does the graph open up or down?

6. For $y = \frac{1}{2}(x - 4)^2 - 5$, does the graph look wider or narrower than $y = x^2$?

7. In $y = a(x - h)^2 + k$, what does a tell you?

8. What happens to the graph if a is negative?

9. What happens to the graph if $|a| > 1$?

Start Tasks – Answers



1. $(1, 3)$

4. $x = 1$

7. The shape /
steepness and
opening

2. $(-2, 1)$

5. **Downward**

8. **It opens downward**

3. $(4, -5)$

6. **Wider**

9. **Narrower**

Start Tasks



10. For $y = 2x^2$, find y when $x = 1$.

11. For $y = 2x^2$, find y when $x = -1$.

12. For $y = 2x^2$, find y when $x = 2$.

13. For $y = -x^2 + 4$, find y when $x = 0$.

14. For $y = -x^2 + 4$, find y when $x = 1$.

15. For $y = -x^2 + 4$, find y when $x = -1$.

16. For $y = \frac{1}{2}(x - 2)^2$, find y when $x = 2$.

17. For $y = \frac{1}{2}(x - 2)^2$, find y when $x = 4$.

18. For $y = \frac{1}{2}(x - 2)^2$, find y when $x = 0$.

Start Tasks – Answers



10. 2

13. 4

16. 0

11. 2

14. 3

17. 2

12. 8

15. 3

18. 2

Start Tasks



19. Which is narrower:
 $y = x^2$ or $y = 3x^2$?

20. Which is wider:
 $y = x^2$ or $y = \frac{1}{4}x^2$?

21. Which opens downward: $y = 2x^2$ or $y = -2x^2$?

22. Write an equation with vertex $(0, 0)$ that is narrower than $y = x^2$.

23. Write an equation with vertex $(0, 0)$ that opens downward.

24. Write an equation with vertex $(3, -2)$ that opens upward and is wider than $y = x^2$.

25. Why do $x = 1$ and $x = 3$ give the same y -value for $y = 2(x - 2)^2 + 1$?

26. For $y = -2(x + 1)^2 + 3$, what is the maximum y -value?

27. For $y = 4(x - 5)^2 - 1$, what is the minimum y -value?

Start Tasks – Answers



19. $y = 3x^2$

22. **Example:** $y = 2x^2$

25. They are both 1
unit from the axis
 $x = 2$

20. $y = \frac{1}{4}x^2$

23. **Example:** $y = -x^2$

26. 3

21. $y = -2x^2$

24. **Example:**
 $y = \frac{1}{2}(x - 3)^2 - 2$

27. -1

Build Tasks



1. For $y = 2(x - 1)^2 - 3$, write the vertex, axis, and opening direction.

2. For $y = -3(x + 2)^2 + 4$, write the vertex, axis, and opening direction.

3. For $y = \frac{1}{2}(x - 5)^2 + 1$, write the vertex, axis, and whether it is wider or narrower than $y = x^2$.

4. Find the y-intercept of $y = 2(x - 3)^2 - 4$.

5. Find the y-intercept of $y = -\frac{1}{2}(x + 2)^2 + 5$.

6. Find the y-intercept of $y = 3(x - 1)^2 + 2$.

7. Find two extra points for $y = 2(x - 1)^2 - 3$, one on each side of the axis.

8. Find two extra points for $y = -3(x + 2)^2 + 4$, one on each side of the

9. Find two extra points for $y = \frac{1}{2}(x - 5)^2 + 1$, one on each side of the

Build Tasks – Answers



1. Vertex $(1, -3)$, axis $x = 1$, opens up

4. 14

7. Example: $(0, -1)$ and $(2, -1)$

2. Vertex $(-2, 4)$, axis $x = -2$, opens down

5. 3

8. Example: $(-3, 1)$ and $(-1, 1)$

3. Vertex $(5, 1)$, axis $x = 5$, wider

6. 5

9. Example: $(3, 3)$ and $(7, 3)$

Build Tasks



10. Solve

$2(x - 1)^2 - 8 = 0$ to
find the x -intercepts.

11. Solve

$-3(x + 2)^2 + 12 = 0$ to
find the x -intercepts.

12. Solve

$\frac{1}{2}(x - 4)^2 - 2 = 0$ to
find the x -intercepts.

13. Does

$y = 2(x - 3)^2 + 1$ cross
the x -axis? Explain.

14. Does

$y = -4(x + 1)^2 - 2$
cross the x -axis?
Explain.

15. Does

$y = -\frac{1}{2}(x - 5)^2 + 6$
cross the x -axis?
Explain.

16. Write the equation
with vertex $(2, -1)$ that
passes through $(3, 1)$.

17. Write the equation
with vertex $(-1, 4)$ that
passes through $(0, 2)$.

18. Write the equation
with vertex $(3, 2)$ that
passes through $(5, 10)$.

Build Tasks – Answers



10. $x = -1, 3$

13. No; minimum is 1

16. $y = 2(x - 2)^2 - 1$

11. $x = -4, 0$

14. No; maximum is -2

17. $y = -2(x + 1)^2 + 4$

12. $x = 2, 6$

**15. Yes; vertex is
above the axis and
opens down**

18. $y = 2(x - 3)^2 + 2$

Build Tasks



19. Which is narrower:
 $y = 2(x - 1)^2$ or
 $y = \frac{1}{2}(x - 1)^2$? Explain.

20. Which has a maximum:
 $y = -2(x + 3)^2 + 1$ or
 $y = 2(x + 3)^2 + 1$?

21. Which has the same vertex as
 $y = 3(x - 4)^2 - 2$:
 $y = -3(x - 4)^2 - 2$ or
 $y = 3(x + 4)^2 - 2$?

22. A graph has vertex $(0, 0)$, opens down, and goes through $(1, -2)$. Write the equation.

23. A graph has vertex $(2, 3)$, opens up, and goes through $(3, 5)$. Write the equation.

24. A graph has vertex $(-4, -1)$, opens up, and goes through $(-2, 7)$. Write the equation.

25. Sketch facts only: for $y = -2(x - 1)^2 + 8$, list the vertex, axis, intercepts, and one

26. Sketch facts only: for $y = \frac{1}{2}(x + 2)^2 - 2$, list the vertex, axis, intercepts, and one

27. Explain why checking one point on each side of the axis helps when graphing

Build Tasks – Answers



19. $y = 2(x - 1)^2$

22. $y = -2x^2$

25. **Vertex** $(1, 8)$, **axis** $x = 1$, **x-ints** $-1, 3$,
example $(0, 6), (2, 6)$

20. $y = -2(x + 3)^2 + 1$

23. $y = 2(x - 2)^2 + 3$

26. **Vertex** $(-2, -2)$,
axis $x = -2$, **x-ints**
 $-4, 0$, **example**
 $(-3, -\frac{3}{2}), (-1, -\frac{3}{2})$

21. $y = -3(x - 4)^2 - 2$

24. $y = 2(x + 4)^2 - 1$

27. **It checks the**
symmetry and shape,
not just the vertex

Push Tasks



1. For $y = 2(x - 1)^2 - 8$, find the vertex, intercepts, and two extra symmetric points.

2. For $y = -3(x + 2)^2 + 12$, find the vertex, intercepts, and two extra symmetric

3. For $y = \frac{1}{2}(x - 4)^2 - 2$, find the vertex, intercepts, and two extra symmetric points.

4. Which is narrower, $y = 4(x - 1)^2$ or $y = (x - 1)^2$? How would that show in a sketch?

5. Which is wider, $y = \frac{1}{4}(x + 3)^2$ or $y = (x + 3)^2$? How would that show in a sketch?

6. Compare $y = 2(x - 1)^2 - 3$ and $y = -2(x - 1)^2 - 3$. What stays the same and what changes?

7. A parabola has vertex $(2, -6)$ and roots -1 and 5 . Write its equation.

8. A parabola has vertex $(-3, 4)$ and passes through $(-1, 0)$. Write its equation.

9. A parabola has vertex $(1, 2)$ and passes through $(3, -6)$. Write its equation.

Push Tasks – Answers



1. Vertex $(1, -8)$, **x-ints** $-1, 3$, **y-int** -6 ,
example
 $(0, -6), (2, -6)$

2. Vertex $(-2, 12)$,
x-ints $-4, 0$, **y-int** 0 ,
example
 $(-3, 9), (-1, 9)$

3. Vertex $(4, -2)$, **x-ints** $2, 6$, **y-int** 6 , **example**
 $(2, 0), (6, 0)$

4. $y = 4(x - 1)^2$; it rises faster so the arms sit closer together

5. $y = \frac{1}{4}(x + 3)^2$; it rises more slowly so the arms spread out

6. Same vertex and axis; one opens up and the other down

7. $y = \frac{2}{3}(x - 2)^2 - 6$

8. $y = -(x + 3)^2 + 4$

9. $y = -2(x - 1)^2 + 2$

Push Tasks



10. A graph has roots -1 and 3 and vertex $(1, -8)$. Write the equation in vertex form.

11. A graph has roots 2 and 6 and vertex $(4, 4)$. Write the equation in vertex form. What does that tell you

12. A graph has roots -5 and -1 and vertex $(-3, 6)$. Write the equation in vertex form.

13. Find a if $y = a(x - 2)^2 - 3$ passes through $(4, 5)$.

14. Find a if $y = a(x + 1)^2 + 4$ passes through $(1, 0)$.

15. Find a if $y = a(x - 3)^2$ passes through $(5, 8)$.

16. For $y = 2(x - 1)^2 - 8$, explain why the graph crosses the x-axis

17. For $y = -3(x + 2)^2 + 12$, explain why the graph has a maximum.

18. For $y = \frac{1}{2}(x - 4)^2 - 2$, explain why the graph is wider than

Push Tasks – Answers



10. $y = 2(x - 1)^2 - 8$

13. $a = 2$

16. The vertex is below the axis and it opens up, so it must cross on both sides

11. $y = -(x - 4)^2 + 4$; it opens down and has a maximum of 4

14. $a = -1$

17. Because $a < 0$, it opens down, so the vertex is the highest point

12. $y = -(x + 3)^2 + 6$

15. $a = 2$

18. Because $|a| = \frac{1}{2} < 1$, the outputs change more slowly away from the vertex

Push Tasks



19. A sketch has vertex $(0, 5)$, opens down, and goes through $(1, 3)$. Write the equation and give the y intercept.

22. Which is steeper near $x = 2$:
 $y = 3(x - 2)^2 + 1$ or
 $y = \frac{1}{3}(x - 2)^2 + 1$?

25. A parabola has axis $x = 3$, opens down, and passes through $(1, 0)$ and $(5, 0)$. Write

20. Vertex $(2, -1)$, up, through $(4, 7)$. Find the equation and x -intercepts.

23. Why do
 $y = -\frac{1}{2}(x + 4)^2 + 6$ and
 $y = -2(x + 4)^2 + 6$
have different roots?

26. A parabola has vertex $(-2, 8)$ and x -intercepts -4 and 0 . Write the equation in

21. A sketch has vertex $(-1, 2)$, opens down, and passes through $(1, -6)$. Write the equation and give one

24. Can changing a alone move the vertex in $y = a(x - h)^2 + k$?

27. A parabola has vertex $(4, -3)$ and goes through $(2, 5)$ and $(6, 5)$. Write the

Push Tasks – Answers



19. $y = -2x^2 + 5$;
y-intercept 5

20. $y = 2(x - 2)^2 - 1$;
x-ints $2 \pm \frac{\sqrt{2}}{2}$

21. $y = -2(x + 1)^2 + 2$;
example extra point
 $(-3, -6)$

22. $y = 3(x - 2)^2 + 1$ **is**
steeper near the
vertex because its a
has larger magnitude

23. Same axis and
vertex because h, k
match; roots differ
because changing a
changes how quickly

24. False when only a
changes; true only if h
or k changes too

25. $y = -2(x - 3)^2 + 8$

26. $y = -2(x + 2)^2 + 8$

27. $y = 2(x - 4)^2 - 3$;
both points are 2 units
from the axis $x = 4$