



Gradient of tangents on curves

Mana Maths

Te reo Māori terms



pātapa

tangent

Open in Te Aka

pārōnaki

gradient

Open in Te Aka

pito hurihuri

turning point

Open in Te Aka

pito tū

stationary point

Open in Te Aka

Gradient of tangents on curves

Gradient of Tangents on Curves



Key idea

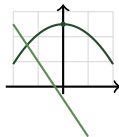
The **gradient** of a curve at a point is the gradient of the **tangent line**. Unlike straight lines, curves have a different gradient at every point.

How to estimate gradient

- Draw the tangent line at the point.
- Pick two grid intersection points on the tangent.
- $m = \frac{\Delta y}{\Delta x}$ (rise over run).
- Horizontal tangent $\Rightarrow m = 0$ (turning point).

Gradient of tangents on curves

Estimating gradient from a tangent



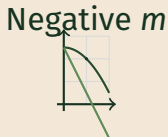
$$\text{Rise} = -4, \text{Run} = 1 \Rightarrow m = -4$$

Gradient of tangents on curves

Gradient of Tangents on Curves



Gradient signs



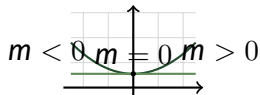
Key idea

A **positive gradient** ($m > 0$) means the curve slopes upward. **Negative** ($m < 0$) means it slopes downward. At a **turning point**, $m = 0$ (horizontal tangent).

Increasing / Decreasing

If $m > 0$, function is **increasing**. If $m < 0$, function is **decreasing**. If $m = 0$, **stationary** (turning point).

Identifying turning points



Gradient changes $- \rightarrow 0 \rightarrow + =$
minimum.

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Stationary point types

Maximum: gradient $+$ $\rightarrow$ 0 $\rightarrow$ $-$
(curve peaks)

Minimum: gradient $-$ $\rightarrow$ 0 $\rightarrow$ $+$
(curve troughs)

Inflection: gradient 0 but sign
unchanged (curve flattens briefly
then continues)

Try these

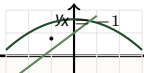
1. A tangent slopes up 2 for 1 right. What is m ?
2. Gradient is $+$ then 0 then $-$. Max or min?
3. At a turning point, what is the gradient?

Answers: 1. $m = 2$ 2. Max 3. $m = 0$

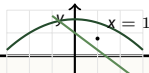
Start Tasks



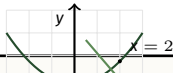
1. Estimate the gradient of the tangent at $x = -1$:



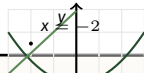
2. Estimate the gradient of the tangent at $x = 1$:



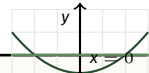
3. Estimate the gradient of the tangent at $x = 2$:



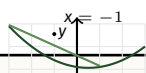
4. Estimate the gradient of the tangent at $x = -2$:



5. Estimate the gradient of the tangent at $x = 0$:



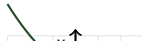
6. Estimate the gradient of the tangent at $x = -1$:



7. Estimate the gradient of the tangent at $x = 1$:



8. Estimate the gradient of the tangent at $x = 2$:



9. Estimate the gradient of the tangent at $x = -2$:



Start Tasks — Answers



1. $m \approx -1.5$

4. $m \approx -1$

7. $m \approx 0.8$

2. $m \approx 1.5$

5. $m = 0$

8. $m \approx -1$

3. $m \approx -2$

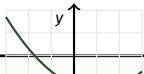
6. $m \approx -0.5$

9. $m \approx 1.8$

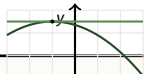
Start Tasks



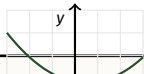
10. Estimate the gradient at the turning point:



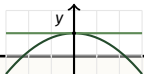
11. Estimate the gradient at the turning point:



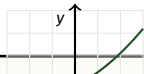
12. Estimate the gradient at the turning point:



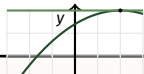
13. Estimate the gradient at the turning point:



14. Estimate the gradient at the turning point:



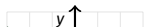
15. Estimate the gradient at the turning point:



16. Is the gradient positive, negative, or zero at the turning point?

17. Is the gradient positive, negative, or zero at the turning point?

18. What is special about the tangent at a turning point?



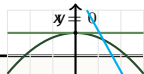
Start Tasks



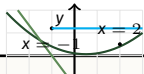
19. Estimate both gradients. Which tangent is steeper?



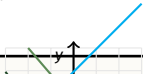
20. Estimate both gradients. Which tangent is steeper?



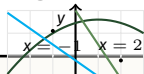
21. Estimate both gradients. Which tangent is steeper?



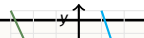
22. Estimate both gradients. Which is more positive?



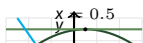
23. Estimate both gradients. Which is more negative?



24. Estimate both gradients. Which has the most negative slope?



25. Estimate both gradients. One is zero — which one?



26. Estimate both gradients. Which absolute gradient is larger?



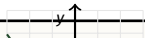
27. Estimate the gradient at $x = -2$ and $x = 1$. Which is steeper?



Build Tasks



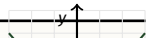
1. Sketch the approximate tangent at $x = -1$ and estimate the gradient:



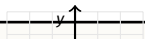
2. Sketch the approximate tangent at $x = 2$ and estimate the gradient:



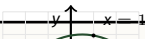
3. Sketch the approximate tangent at $x = 0$ and estimate the gradient:



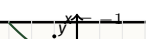
4. Sketch the approximate tangent at $x = -2$ and estimate the gradient:



5. Sketch the approximate tangent at $x = 1$ and estimate the gradient:



6. Sketch the approximate tangent at $x = -1$ and estimate the gradient:



7. Sketch the tangent at $x = -2$ and $x = 2$ on this cubic. Estimate both gradients:

8. Sketch the tangent at $x = -1$ and $x = 1$ on this cubic. Estimate both gradients:

9. Sketch the tangent at $x = 0$ on this cubic and estimate the gradient:

Build Tasks — Answers



1. $m \approx -1.5$

4. $m \approx -1$

7. $x = -2: m \approx -1,$
 $x = 2: m \approx 1$

2. $m \approx -0.5$

5. $m \approx 0$

8. $x = -1: m \approx 0.5,$
 $x = 1: m \approx -0.5$

3. $m = 0$

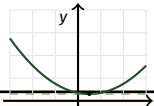
6. $m \approx -0.7$

9. $m \approx -0.3$

Build Tasks



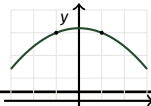
10. For what x values is this graph increasing?



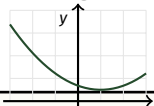
11. For what x values is this graph decreasing?



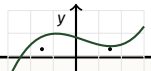
12. For what x is the gradient positive?



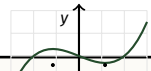
13. For what x is the gradient negative?



14. State the interval(s) where this function is increasing:



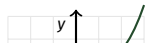
15. State the interval(s) where this function is decreasing:



16. State where $f'(x) > 0$ (gradient positive):



17. State where $f'(x) < 0$ (gradient negative):



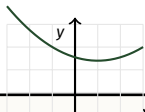
18. List the x -values where $f'(x) = 0$:



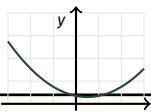
Build Tasks



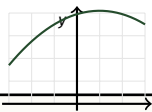
19. Estimate x where the gradient $m = 0$:



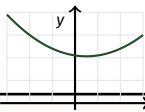
20. Estimate x where the gradient $m \approx 2$:



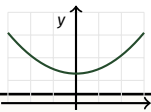
21. Estimate x where the gradient $m \approx -1$:



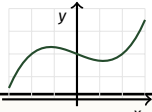
22. Estimate x where the gradient $m \approx 3$:



23. Estimate x where the gradient $m \approx -2$:



24. Estimate values of x where $m \approx -1$:



25. Estimate x where $m = 0$ on this cubic:



26. Estimate x where $m \approx 1$ on this cubic:



27. Find two x -values where $m \approx 0.5$:



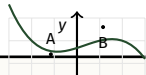
Push Tasks



1. For this curve, state the gradient at each turning point. Is it a maximum or minimum?

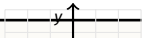


2. State the gradient at A and B. Which is a maximum, which is a minimum?



3. Identify the turning points and state their gradient:

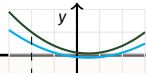
4. Compare the gradient at $x = -2$ on both curves. Which is steeper?



5. At $x = 2$, which curve has the larger gradient?



6. Which curve has the steeper positive gradient at $x = -2$?



7. State the gradient at each turning point and classify as max or min:

8. Find the stationary points and their types:



9. At turning points A and B, what is the gradient? Which is max/min?

Push Tasks — Answers



1. **A:** $m = 0$, max; **B:**
 $m = 0$, min

2. **A:** $m = 0$, min; **B:**
 $m = 0$, max

3. **A:** $m = 0$, min; **B:**
 $m = 0$, max

4. The green
(narrower) curve is
steeper

5. The green
(narrower) curve has
larger gradient

6. The green curve
(steeper)

7. **A:** $m = 0$, max; **B:**
 $m = 0$, min

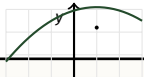
8. **A:** $m = 0$, max; **B:**
 $m = 0$, min

9. Both $m = 0$; **A** is
max, **B** is min

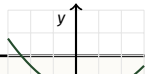
Push Tasks



10. Describe how the gradient changes as x increases from -3 to 3 :



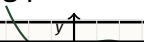
11. Describe the gradient behaviour for this quadratic:



12. Trace the sign of the gradient from left to right:



13. Describe the gradient sign on each interval separated by turning points:

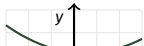


14. Where is the gradient most positive? Most negative?



15. Complete: "As $x \rightarrow -3$, the gradient is ____; as x increases, the gradient ____ until at the turning point it is ____"

16. Sketch a gradient sign diagram for this curve:



17. For the cubic shown, describe how $f'(x)$ changes sign across the domain:

18. Explain why the gradient at $x = -1$ is different to $x = 1$ for this curve:

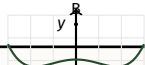
Push Tasks



19. Identify all stationary points and classify each as max, min, or inflection:



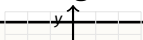
20. This quartic has three stationary points. Classify each:



21. The cubic has a point of inflection at A. What is the gradient there?



22. Where are the stationary points? A is a point of inflection — what is the gradient?



23. Classify points A, B, and C. Is the gradient zero at all three?



24. This curve has a maximum at A and an inflection point at B. State the gradient values:

25. Find the x-coordinates where the gradient is zero. Classify each:

26. This quartic has a minimum at A, maximum at B, minimum at C. What is

27. Analyse this curve: find all stationary points, state gradient at each, and classify: