Te reo Māori terms

hikuaua whakareatanga
exponential pattern
Open in Te Aka

kauwhata
graph
Open in Te Aka

tepe kōpiko
asymptote
Open in Te Aka

taupū
exponent
Open in Te Aka

Exponential Patterns

Key idea

An exponential pattern **multiplies** by the same factor each step — a constant ratio.

Rule: $y = a \cdot b^x$.

Growth or decay

$b > 1$ gives **growth**.

$0 < b < 1$ gives **decay**.

Example

$4,\ 12,\ 36,\ 108, \ldots$ multiplies by 3 each time.

Starting value 4, so $y = 4 \cdot 3^x$.

Common mistake

A constant *difference* means **linear**, not exponential.

Only a constant *ratio* is exponential.

= copy this into your book. Boxes without a pencil are just to read.

Exponential Graphs

Key idea

In $y = a \cdot b^x + c$, the $+c$ slides the graph so the horizontal asymptote is $y = c$.

The graph gets close to $y = c$ but never reaches it.

Growth example

$y = 2^x + 3$: shifted up 3.

y-intercept $= 4$, asymptote $y = 3$.

Decay example

$y = 6\left(\frac{1}{2}\right)^x + 1$: halves each step.

y-intercept $= 7$, asymptote $y = 1$.

Common mistake

The shift c changes the asymptote; the stretch a does **not**.

Keep those two effects separate when sketching.

= copy this into your book. Boxes without a pencil are just to read.

Start Tasks

1. Continue:
$3, 6, 12, 24, \ldots$

2. Continue:
$5, 10, 20, 40, \ldots$

3. Continue:
$81, 27, 9, 3, \ldots$

4. Continue:
$2, 6, 18, 54, \ldots$

5. Continue:
$64, 32, 16, 8, \ldots$

6. Continue:
$7, 14, 28, 56, \ldots$

7. What is the multiplier in $4, 8, 16, 32, \ldots$?

8. What is the multiplier in $90, 30, 10, \frac{10}{3}, \ldots$?

9. Is $2, 5, 8, 11, \ldots$ exponential? Explain briefly.

Start Tasks — Answers

1.
$48, 96$

2.
$80, 160$

3.
$1, \frac{1}{3}$

4.
$162, 486$

5.
$4, 2$

6.
$112, 224$

7.
Multiply by 2.

8.
Multiply by $\frac{1}{3}$.

9.
No, it is linear with difference $+3$.

Start Tasks

10. Fill the table for $y = 2^x$ when $x = 0, 1, 2, 3$.

11. Fill the table for $y = 3^x$ when $x = 0, 1, 2, 3$.

12. Fill the table for $y = \left(\frac{1}{2}\right)^x$ when $x = 0, 1, 2, 3$.

13. A table gives $y = 5, 10, 20, 40$. Write the next two y-values.

14. A table gives $y = 81, 27, 9, 3$. Is this growth or decay?

15. For $y = 2^x$, state the y-intercept and horizontal asymptote.

16. For $y = 2^x + 3$, state the y-intercept and horizontal asymptote.

17. For $y = \left(\frac{1}{2}\right)^x - 1$, state the y-intercept and horizontal

18. For $y = 2^x$, write the first four coordinate pairs from $x = 0$ to $x = 3$.

Start Tasks — Answers

10.
$1, 2, 4, 8$

11.
$1, 3, 9, 27$

12.
$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

13.
$80, 160$

14.
Decay.

15.
***y*-intercept** 1**,**
asymptote $y = 0$**.**

16.
***y*-intercept** 4**,**
asymptote $y = 3$**.**

17.
***y*-intercept** 0**,**
asymptote $y = -1$**.**

18.
$(0, 1), (1, 2), (2, 4), (3, 8)$

Start Tasks

19. For $y = 3 \cdot 2^x$, write the coordinates for $x = 0, 1, 2$.

20. For $y = 5\left(\frac{1}{2}\right)^x$, write the coordinates for $x = 0, 1, 2$.

21. By $x = 3$, which is larger: 2^x or 3^x?

22. Classify $2, 4, 8, 16, \ldots$ as linear, quadratic, or exponential.

23. Classify $5, 8, 11, 14, \ldots$ as linear, quadratic, or exponential.

24. Classify $1, 4, 9, 16, \ldots$ as linear, quadratic, or exponential.

25. Does $y = 2^x$ ever cross the x-axis?

26. As x increases, does $y = \left(\frac{1}{2}\right)^x$ rise or fall?

27. Give three points on the graph $y = 2^x + 1$.

Start Tasks — Answers

19.
$(0, 3), (1, 6), (2, 12)$

20.
$(0, 5), (1, 2.5), (2, 1.25)$

21.
$3^3 = 27$ **is larger than** $2^3 = 8$**.**

22.
Exponential.

23.
Linear.

24.
Quadratic.

25.
No, it stays above the *x*-axis.

26.
It falls.

27.
For example:
$(0, 2), (1, 3), (2, 5)$**.**

Build Tasks

1. Fill the table for $y = 2^{x-1}$ when $x = 0, 1, 2, 3$.

2. Fill the table for $y = 3^x + 2$ when $x = 0, 1, 2, 3$.

3. Fill the table for $y = 4\left(\frac{1}{2}\right)^x$ when $x = 0, 1, 2, 3$.

4. Fill the table for $y = 2 \cdot 3^x - 1$ when $x = 0, 1, 2, 3$.

5. Write an exponential rule for $3, 6, 12, 24, \ldots$ with $x = 0, 1, 2, \ldots$.

6. Write an exponential rule for $80, 40, 20, 10, \ldots$ with $x = 0, 1, 2, \ldots$.

7. Write an exponential rule for $5, 15, 45, 135, \ldots$ with $x = 0, 1, 2, \ldots$.

8. Write an exponential rule for $6, 12, 24, 48, \ldots$ with $x = 0, 1, 2, \ldots$.

9. For $y = 3^x - 4$, state growth/decay and the horizontal asymptote.

Build Tasks — Answers

1.
$\frac{1}{2}, 1, 2, 4$

2.
$3, 5, 11, 29$

3.
$4, 2, 1, \frac{1}{2}$

4.
$1, 5, 17, 53$

5.
$y = 3 \cdot 2^x$

6.
$y = 80\left(\frac{1}{2}\right)^x$

7.
$y = 5 \cdot 3^x$

8.
$y = 6 \cdot 2^x$

9.
Growth, asymptote
$y = -4$.

Build Tasks

10. For $y = 5\left(\frac{1}{2}\right)^x + 2$, state growth/decay and the horizontal asymptote.

11. For $y = 2 \cdot 4^x$, state growth/decay and the horizontal asymptote.

12. Find the y-intercept of $y = 2^{x+1} + 3$.

13. Find the y-intercept of $y = 3\left(\frac{1}{2}\right)^x - 2$.

14. Find the y-intercept of $y = 4^x - 5$.

15. Compare $y = 2^x$ and $y = 2^x + 3$. What changes?

16. Compare $y = 2^x$ and $y = 3 \cdot 2^x$. What changes?

17. Compare $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$. What changes?

18. A table gives $y = 4, 7, 13, 25$. Write the rule in terms of x.

Build Tasks — Answers

10.
Decay, asymptote $y = 2$.

11.
Growth, asymptote $y = 0$.

12.
5

13.
1

14.
-4

15.
Same shape, shifted up 3.

16.
Vertical stretch by 3; **same asymptote.**

17.
Growth changes to decay.

18.
$y = 3 \cdot 2^x + 1$

Build Tasks

19. A table gives $y = 9, 5, 3, 2$. Write the rule in terms of x.

20. A table gives $y = -1, 1, 5, 13$. Write the rule in terms of x.

21. For $y = 2^x - 3$, is there an x-intercept? If so, between which integers?

22. For $y = \left(\frac{1}{2}\right)^x + 2$, is there an x-intercept? Explain.

23. Why is the horizontal asymptote of $y = 3^x$ equal to $y = 0$?

24. Give three points and the asymptote for $y = 2^{x-2} + 1$.

25. Give three points and the asymptote for $y = 3\left(\frac{1}{2}\right)^x - 1$.

26. Give three points and the asymptote for $y = 2 \cdot 3^x + 4$.

27. Which grows faster by $x = 4$: 2^x or $2x + 1$?

Build Tasks — Answers

19.

$y = 8\left(\frac{1}{2}\right)^x + 1$

20.

$y = 2 \cdot 2^x - 3$

21.

Yes, between 1 and 2.

22.

No, the graph stays above $y = 2$.

23.

Because $3^x \to 0$ as $x \to -\infty$.

24.

For example:
$(2, 2), (3, 3), (4, 5)$**, asymptote $y = 1$.**

25.

For example:
$(0, 2), (1, \frac{1}{2}), (2, -\frac{1}{4})$**, asymptote** $y = -\frac{1}{2}$**.**

26.

For example:
$(0, 6), (1, 10), (2, 22)$**, asymptote $y = 4$.**

27.

2^x grows faster.

Push Tasks

1. A table gives $y = 2, 5, 11, 23$. Write the exponential rule.

2. A table gives $y = 7, 4, 2.5, 1.75$. Write the exponential rule.

3. A table gives $y = 0, 2, 8, 26$. Write the exponential rule.

4. Why does the ratio test work for $5, 15, 45, 135, \ldots$ but the difference test does not?

5. Do $y = 2^x + 1$ and $y = 2^{x+1}$ have the same asymptote? Explain.

6. Compare $y = 2^x - 3$ and $y = 2 \cdot 2^x - 3$. What stays the same and what changes?

7. Find a if $y = a \cdot 2^x + 1$ passes through $(2, 9)$.

8. Find a if $y = a\left(\frac{1}{2}\right)^x - 3$ passes through $(0, 5)$.

9. Find a if $y = a \cdot 3^x + 2$ passes through $(2, 29)$.

Push Tasks — Answers

1.

$y = 3 \cdot 2^x - 1$

2.

$y = 6\left(\frac{1}{2}\right)^x + 1$

3.

$y = 3^x - 1$

4.

Ratios stay constant at 3, differences do not.

5.

No. They are $y = 1$ and $y = 0$.

6.

Same asymptote $y = -3$; the second is steeper with larger y-values.

7.

$a = 2$

8.

$a = 8$

9.

$a = 3$

Push Tasks

10. A graph starts at 6 when $x = 0$ and triples each step. Write the rule.

11. A graph starts at 10 when $x = 0$ and halves each step. Write the rule.

12. Use the shape of 2^x: stretch by 4 and move down 5. Write the rule.

13. For $y = 2^x + 4$, state the asymptote and whether an x-intercept exists.

14. For $y = 3\left(\frac{1}{2}\right)^x - 4$, state the asymptote and whether an x-intercept exists.

15. For $y = 5 \cdot 2^x - 7$, estimate the interval containing the x-intercept.

16. State the y-intercept, asymptote, and growth/decay for

17. State the y-intercept, asymptote, and growth/decay for

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Push Tasks — Answers

10.
$y = 6 \cdot 3^x$

11.
$y = 10\left(\frac{1}{2}\right)^x$

12.
$y = 4 \cdot 2^x - 5$

13.
Asymptote $y = 4$; no x-intercept.

14.
Asymptote $y = -4$; yes, one x-intercept.

15.
Between 0 and 1.

16.
y-intercept -1, asymptote $y = -2$, growth.

17.
y-intercept 7, asymptote $y = 1$, decay.

18.
y-intercept -2.5, asymptote $y = -3$, growth.

Push Tasks

19. A table gives $y = 3, 9, 27, 81$. Write the rule, y-intercept, and asymptote.

20. A table gives $y = 13, 7, 4, 2.5$. Write the rule, y-intercept, and asymptote.

21. A table gives $y = -1, 5, 17, 41$. Write the rule, y-intercept, and asymptote.

22. For $x > 0$, which is steeper: 2^x or $2 \cdot 2^x$? Why?

23. Why can $y = \left(\frac{1}{2}\right)^x + 3$ never go below 3?

24. Why does changing the base from 2 to $\frac{1}{2}$ turn growth into decay?

25. A pattern is $4, 12, 36, 108, \ldots$. Write the rule and find the term when $x = 5$.

26. A pattern follows $y = a \cdot 2^x - 1$ and passes through $(3, 23)$. Find a.

27. Write one possible exponential rule with asymptote $y = -2$ and y-intercept 3.

Push Tasks — Answers

19.
$y = 3^{x+1}$ **or** $3 \cdot 3^x$**;**
y**-intercept** 3**;**
asymptote $y = 0$**.**

20.
$y = 12\left(\frac{1}{2}\right)^x + 1$**;**
y**-intercept** 13**;**
asymptote $y = 1$**.**

21.
$y = 6 \cdot 2^x - 7$**;**
y**-intercept** -1**;**
asymptote $y = -7$**.**

22.
$2 \cdot 2^x$**, because it is a vertical stretch of** 2^x**.**

23.
Because $\left(\frac{1}{2}\right)^x > 0$**, so adding** 3 **keeps it above** 3**.**

24.
A base between 0 **and** 1 **makes each step smaller instead of larger.**

25.
$y = 4 \cdot 3^x$**; when** $x = 5$**,** $y = 972$**.**

26.
$a = 3$

27.
For example:
$y = 5 \cdot 2^x - 2$**.**