Distance between two points (Pythagoras)

Mana Maths

Te reo Māori terms

taunga
coordinate
Open in Te Aka

ture a Pythagoras
Pythagoras' theorem
Open in Te Aka

tapatoru hāngai
right-angle triangle
Open in Te Aka

tāroa
hypotenuse
Open in Te Aka

Distance Between Two Points

Key idea

The horizontal change Δx and vertical change Δy are the two shorter sides of a right-angled triangle:

$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$.

Method

Δx = gap in the x-values.
Δy = gap in the y-values.
Then $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$.

Example

From $(0, 0)$ to $(3, 4)$:

$\Delta x = 3,\ \Delta y = 4$
$d^2 = 3^2 + 4^2 = 25$
$d = 5$

Common mistake

Do not use the coordinates as the side lengths.
Use the **changes** between them: Δx and Δy.

Harder Cases - Notes and Steps

Negative coordinates

From $(-2, -3)$ to $(3, 9)$:

$\Delta x = 3 - (-2) = 5$

$\Delta y = 9 - (-3) = 12$

$d^2 = 5^2 + 12^2 = 169$, so $d = 13$.

Not always a whole number

From $(1, 1)$ to $(5, 5)$:

$\Delta x = 4,\ \Delta y = 4$

$d = \sqrt{32} = 4\sqrt{2}$

Common mistake

With negatives, subtract carefully.

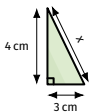
Δx is how far apart the x-values are, so d is always positive.

= copy this into your book. Boxes without a pencil are just to read.

Start Tasks

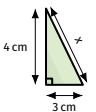
Find the horizontal change, vertical change, and distance using Pythagoras' theory.

1.



From $(0, 0)$ to $(3, 4)$, find the horizontal change, the vertical change, and the distance.

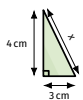
2.



From $(1, 2)$ to $(4, 6)$, find the distance.

Start Tasks — Answers

1. From $(0, 0)$ to $(3, 4)$, find the horizontal change, the vertical change, and the distance.



Solution

$\Delta x = 3,\ \Delta y = 4$

$d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

2. From $(1, 2)$ to $(4, 6)$, find the distance.

Solution

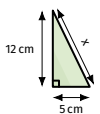
$\Delta x = 3,\ \Delta y = 4$

$d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Start Tasks

Find the horizontal change, vertical change, and distance using Pythagoras' theory.

3.



From $(2, 1)$ to $(7, 13)$, find the distance.

4. From $(2, 3)$ to $(8, 11)$, find the distance.

Start Tasks — Answers

3. From $(2, 1)$ to $(7, 13)$, find the distance.

Solution

$\Delta x = 5,\ \Delta y = 12$

$d = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

4. From $(2, 3)$ to $(8, 11)$, find the distance.

Solution

$\Delta x = 6,\ \Delta y = 8$

$d = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

Start Tasks

Find the distance between two points, including cases with negative coordinates.

5. From $(4, 5)$ to $(11, 29)$, find the distance.

6. From $(-1, 1)$ to $(2, 5)$, find the distance.

7. From $(-2, -3)$ to $(3, 9)$, find the distance.

8. From $(-3, -1)$ to $(4, 23)$, find the distance.

Start Tasks — Answers

5. From $(4, 5)$ to $(11, 29)$, find the distance.

Solution

$\Delta x = 7,\ \Delta y = 24$

$d = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$

6. From $(-1, 1)$ to $(2, 5)$, find the distance.

Solution

$\Delta x = 3,\ \Delta y = 4$

$d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

7. From $(-2, -3)$ to $(3, 9)$, find the distance.

Solution

$\Delta x = 5,\ \Delta y = 12$

$d = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

8. From $(-3, -1)$ to $(4, 23)$, find the distance.

Solution

$\Delta x = 7,\ \Delta y = 24$

$d = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$

Start Tasks

Identify the Pythagorean triple that helps, then find the distance.

9. From $(1, -2)$ to $(9, 13)$, find the distance.

10. Two points are $(0, 0)$ and $(12, 16)$. Which Pythagorean triple helps? Then find the distance.

11. Two points are $(3, 7)$ and $(10, 31)$. Which Pythagorean triple helps? Then find the distance.

12. Two points are $(-6, 2)$ and $(2, 17)$. Which Pythagorean triple helps? Then find the distance.

Start Tasks — Answers

9. From $(1, -2)$ to $(9, 13)$, find the distance.

Solution

$\Delta x = 8,\ \Delta y = 15$

$d = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$

10. Two points are $(0, 0)$ and $(12, 16)$. Which Pythagorean triple helps? Then find the distance.

Solution

$\Delta x = 12,\ \Delta y = 16$: the $3:4:5$ triple $\times 4$

$d = 20$

11. Two points are $(3, 7)$ and $(10, 31)$. Which Pythagorean triple helps? Then find the distance.

Solution

$\Delta x = 7,\ \Delta y = 24$: the $7:24:25$ triple

$d = 25$

12. Two points are $(-6, 2)$ and $(2, 17)$. Which Pythagorean triple helps? Then find the distance.

Solution

$\Delta x = 8,\ \Delta y = 15$: the $8:15:17$ triple

$d = 17$

Build Tasks

Find the distance and show it equals a given value.

1. Find the distance between $(-1, 2)$ and $(5, 10)$.

2. Find the distance between $(-4, -3)$ and $(8, 2)$.

3. Show that the distance from $(2, 1)$ to $(10, 7)$ is 10.

4. Show that the distance from $(-3, -4)$ to $(2, 8)$ is 13.

Build Tasks — Answers

1. Find the distance between $(-1, 2)$ and $(5, 10)$.

Solution

$\Delta x = 6,\ \Delta y = 8$

$d = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

2. Find the distance between $(-4, -3)$ and $(8, 2)$.

Solution

$\Delta x = 12,\ \Delta y = 5$

$d = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

3. Show that the distance from $(2, 1)$ to $(10, 7)$ is 10.

Solution

$\Delta x = 8,\ \Delta y = 6$

$d = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ – confirmed

4. Show that the distance from $(-3, -4)$ to $(2, 8)$ is 13.

Solution

$\Delta x = 5,\ \Delta y = 12$

$d = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ – confirmed

Build Tasks

Find a missing coordinate given the distance, or identify an error in a method.

5. One point is $(1, 1)$. Another point is $(x, 5)$. If the distance is 5, find a possible value of x.

6. One point is $(-2, 3)$. Another point is $(4, y)$. If the distance is 10, find a possible value of y.

7. A student says the distance from $(1, 2)$ to $(4, 6)$ is $3 + 4 = 7$. Explain the mistake.

8. A student says the distance from $(-3, 1)$ to $(5, 16)$ is $8^2 + 15^2 = 289$. What did they forget?

Build Tasks — Answers

5. One point is $(1, 1)$. Another point is $(x, 5)$. If the distance is 5, find a possible value of x.

Solution

$\Delta y = 4$; $\Delta x^2 + 16 = 25 \Rightarrow \Delta x = \pm 3$

$x = 4$ (or $x = -2$)

6. One point is $(-2, 3)$. Another point is $(4, y)$. If the distance is 10, find a possible value of y.

Solution

$\Delta x = 6$; $36 + \Delta y^2 = 100 \Rightarrow \Delta y = \pm 8$

$y = 11$ (or $y = -5$)

7. A student says the distance from $(1, 2)$ to $(4, 6)$ is $3 + 4 = 7$. Explain the mistake.

Solution

Correct: $d = \sqrt{3^2 + 4^2} = 5 \neq 7$.

8. A student says the distance from $(-3, 1)$ to $(5, 16)$ is $8^2 + 15^2 = 289$. What did they forget?

Solution

They forgot to take the square root: $d = \sqrt{289} = 17$

Build Tasks

Compare segment lengths, and verify whether a distance is a whole number.

9. Which is longer: the segment from $(0, 0)$ to $(8, 15)$ or the segment from $(0, 0)$ to $(7, 24)$?

10. Which is longer: the segment from $(2, 3)$ to $(8, 11)$ or the segment from $(2, 3)$ to $(7, 15)$?

11. The distance between $(-1, -1)$ and $(7, 5)$ is a whole number. Show why.

12. The distance between $(-4, 2)$ and $(1, 14)$ is a whole number. Show why.

Build Tasks — Answers

9. Which is longer: the segment from $(0, 0)$ to $(8, 15)$ or the segment from $(0, 0)$ to $(7, 24)$?

Solution

$d_1 = \sqrt{8^2 + 15^2} = 17$

$d_2 = \sqrt{7^2 + 24^2} = 25$ — d_2 is longer.

10. Which is longer: the segment from $(2, 3)$ to $(8, 11)$ or the segment from $(2, 3)$ to $(7, 15)$?

Solution

$d_1 = \sqrt{6^2 + 8^2} = 10$

$d_2 = \sqrt{5^2 + 12^2} = 13$ — d_2 is longer.

11. The distance between $(-1, -1)$ and $(7, 5)$ is a whole number. Show why.

Solution

$\Delta x = 8, \Delta y = 6$; $d = \sqrt{100} = 10$, whole.

12. The distance between $(-4, 2)$ and $(1, 14)$ is a whole number. Show why.

Solution

$\Delta x = 5, \Delta y = 12$; $d = \sqrt{169} = 13$, whole.

Push Tasks

Find the distance, including the diagonal of a shape.

1. Find the distance between $(-6, -4)$ and $(9, 16)$.

2. A square has vertices $(1, 1)$, $(7, 1)$, $(7, 7)$, $(1, 7)$. Find the diagonal length.

3. A rectangle has corners $(-2, -1)$, $(6, -1)$, $(6, 14)$, $(-2, 14)$. Find the diagonal length.

4. Point $A = (2, 3)$. Find a point B so that $AB = 13$ and both coordinates of B are integers.

Push Tasks — Answers

1. Find the distance between $(-6, -4)$ and $(9, 16)$.

Solution

$\Delta x = 15,\ \Delta y = 20$

$d = \sqrt{15^2 + 20^2} = \sqrt{625} = 25$

2. A square has vertices $(1, 1)$, $(7, 1)$, $(7, 7)$, $(1, 7)$. Find the diagonal length.

Solution

$\Delta x = 6,\ \Delta y = 6$

$d = \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2} \approx 8.49$

3. A rectangle has corners $(-2, -1)$, $(6, -1)$, $(6, 14)$, $(-2, 14)$. Find the diagonal length.

Solution

$\Delta x = 8,\ \Delta y = 15$

$d = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$

4. Point $A = (2, 3)$. Find a point B so that $AB = 13$ and both coordinates of B are integers.

Solution

Using the $5:12:13$ triple:

$B = (2 + 5, 3 + 12) = (7, 15)$

Push Tasks

Construct an integer-coordinate point at a given distance from another point.

5. For $A = (-4, 5)$, give an integer-coordinate point B with $AB = 17$.

6. The distance from $(-3, 2)$ to $(x, 14)$ is 13. Find both values of x.

7. A segment is 15 units long with horizontal change 9. Find both possible vertical changes.

8. A segment is 26 units long with vertical change 24. Find both possible horizontal changes.

Push Tasks — Answers

5. For $A = (-4, 5)$, give an integer-coordinate point B with $AB = 17$.

Solution

Using the $8:15:17$ triple:

$B = (-4 + 8, 5 + 15) = (4, 20)$

6. The distance from $(-3, 2)$ to $(x, 14)$ is 13. Find both values of x.

Solution

$\Delta y = 12$; $\Delta x^2 + 144 = 169 \Rightarrow \Delta x = \pm 5$

$x = 2$ or $x = -8$

7. A segment is 15 units long with horizontal change 9. Find both possible vertical changes.

Solution

$81 + v^2 = 225 \Rightarrow v^2 = 144$

$v = 12$ or $v = -12$

8. A segment is 26 units long with vertical change 24. Find both possible horizontal changes.

Solution

$h^2 + 576 = 676 \Rightarrow h^2 = 100$

$h = 10$ or $h = -10$

Push Tasks

Reason about and verify distance results.

9. For $(-2, -3)$ and $(3, 9)$, explain why $5^2 + 12^2 = 13^2$ gives the distance.

10. A student claims $(1, 1)$ and $(6, 13)$ are 14 units apart. Test the claim.

11. Give two points an irrational distance apart. Show the exact distance.

12. Create two points with integer coordinates that are 25 units apart.

Push Tasks — Answers

9. For $(-2, -3)$ and $(3, 9)$, explain why $5^2 + 12^2 = 13^2$ gives the distance.

Solution

$\Delta x = 5, \Delta y = 12$ are the legs; the distance is the hypotenuse $d = 13$.

10. A student claims $(1, 1)$ and $(6, 13)$ are 14 units apart. Test the claim.

Solution

$\Delta x = 5, \Delta y = 12$; $d = \sqrt{169} = 13 \neq 14$: false.

11. Give two points an irrational distance apart. Show the exact distance.

Solution

E.g. $(0, 0)$ and $(1, 2)$:
$d = \sqrt{1^2 + 2^2} = \sqrt{5}$ (irrational)

12. Create two points with integer coordinates that are 25 units apart.

Solution

E.g. using the $7:24:25$ triple:
$(0, 0)$ and $(7, 24)$: $d = \sqrt{49 + 576} = 25$