



Differentiating power functions

Mana Maths

Te reo Māori terms



ture pū

power rule

Open in Te Aka

pārōnaki

derivative

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**pārōnaki
kupu-kupu**

differentiate term-by-term

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**tuhituhinana
Leibniz**

leibniz notation

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The Power Rule



Key idea



If $f(x) = x^n$ then
 $f'(x) = nx^{n-1}$.

Multiply by the exponent, then
drop the exponent by one.

With a coefficient



If $f(x) = ax^n$ then $f'(x) = anx^{n-1}$.
 $f(x) = 5x^4 \rightarrow f'(x) = 20x^3$

Common mistake

Multiply by the coefficient too.
For $f(x) = 3x^4$: $f'(x) = 3 \times 4 \times x^3 = 12x^3$, **not** $4x^3$.



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Differentiate Term by Term



Key idea



Differentiation is linear: differentiate each term on its own and add the results.

A constant differentiates to 0.

Example



$$f(x) = 3x^4 + 6x^3 - 7x^2 - 2x + 3$$

$$f'(x) = 12x^3 + 18x^2 - 14x - 2$$

(The +3 differentiates to 0.)

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Rewrite as Powers First



Key idea

Write fractions and roots as powers of x first, then use the power rule:

$$\frac{1}{x} = x^{-1} \quad \frac{1}{x^2} = x^{-2}$$

$$\sqrt{x} = x^{\frac{1}{2}}$$

Fractions

$$f(x) = 3x^2 + \frac{2}{x} + 5$$

$$= 3x^2 + 2x^{-1} + 5$$

$$f'(x) = 6x - 2x^{-2} = 6x - \frac{2}{x^2}$$

Roots

$$f(x) = 4\sqrt{x} = 4x^{\frac{1}{2}}$$

$$f'(x) = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$$



Simplify First: Products & Quotients



Expand first



$$f(x) = (2x - 1)(x + 3)$$

$$= 2x^2 + 5x - 3$$

$$f'(x) = 4x + 5$$

Divide through first



$$f(x) = \frac{4x^3 + 7x^2 - 6x - 5}{x^2}$$

$$= 4x + 7 - 6x^{-1} - 5x^{-2}$$

$$f'(x) = 4 + \frac{6}{x^2} + \frac{10}{x^3}$$

Common mistake

Divide *every* term by the denominator: $(x^3 + 2x)/x = x^2 + 2$, **not** $x^3 + 2$.



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Start Tasks



1. $f(x) = x^2$. Find $f'(x)$.

2. $f(x) = x^3$. Find $f'(x)$.

3. $f(x) = x^4$. Find $f'(x)$.

4. $f(x) = 2x^2$. Find $f'(x)$.

5. $f(x) = 3x^3$. Find $f'(x)$.

6. $f(x) = 5x^4$. Find $f'(x)$.

7. $f(x) = x^2 + 3x$. Find $f'(x)$.

8. $f(x) = 2x^2 + 5x$. Find $f'(x)$.

9. $f(x) = 3x^3 + x^2$. Find $f'(x)$.

Start Tasks — Answers



1. $f'(x) = 2x$

4. $f'(x) = 4x$

7. $f'(x) = 2x + 3$

2. $f'(x) = 3x^2$

5. $f'(x) = 9x^2$

8. $f'(x) = 4x + 5$

3. $f'(x) = 4x^3$

6. $f'(x) = 20x^3$

9. $f'(x) = 9x^2 + 2x$

Start Tasks



10. $f(x) = 6x^2 + 9x - 3$.
Find $f'(x)$.

11. $f(x) = 4x^2 + 7x - 2$.
Find $f'(x)$.

12. $f(x) = 5x^2 - 3x + 1$.
Find $f'(x)$.

13. $f(x) =$
 $3x^4 + 6x^3 - 7x^2 - 2x + 3$.
Find $f'(x)$.

14. $f(x) =$
 $2x^4 + 5x^3 - 8x^2 + 3x - 1$.
Find $f'(x)$.

15. $f(x) =$
 $4x^4 - 3x^3 + 6x^2 - 5x + 2$.
Find $f'(x)$.

16.
 $f(x) = 5x^4 - 3x^2 + 6x - 9$.
Find $f'(x)$.

17. $f(x) = 7$ (constant).
Find $f'(x)$.

18. $f(x) = 4x^5 + 8$. Find
 $f'(x)$.

Start Tasks — Answers



10. $f'(x) = 12x + 9$

13. $f'(x) =$
 $12x^3 + 18x^2 - 14x - 2$

16. $f'(x) = 20x^3 - 6x + 6$

11. $f'(x) = 8x + 7$

14. $f'(x) =$
 $8x^3 + 15x^2 - 16x + 3$

17. $f'(x) = 0$

12. $f'(x) = 10x - 3$

15. $f'(x) =$
 $16x^3 - 9x^2 + 12x - 5$

18. $f'(x) = 20x^4$

Start Tasks



19.

$$f(x) = 2x^7 + 5x^4 - 3x^2.$$

Find $f'(x)$.

20. $f(x) = x^8 - 2x^5 + 7x.$
Find $f'(x)$.

21.

$$f(x) = 3x^6 - 4x^3 + 9x^2.$$

Find $f'(x)$.

22.

$$f(x) = \frac{1}{2}x^4 + \frac{3}{4}x^2 - x.$$

Find $f'(x)$.

23. $f(x) =$

$$\frac{4}{5}x^4 - \frac{4}{5}x^3 + 3x^2 - 9.$$

Find $f'(x)$.

24.

$$f(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - 5x + 1.$$

Find $f'(x)$.

25.

$$f(x) = 0.5x^5 - 2.4x^4 - 3.5x^3 + 1.6x^2 - 4.$$

Find $f'(x)$.

26.

$$f(x) = 1.2x^4 + 0.8x^3 - 2.5x^2 + 3.1x - 0.5.$$

Find $f'(x)$.

27.

$$f(x) = 3.6x^5 - 1.2x^4 + 2.8x^3 - 4.3x^2 + 6.$$

Find $f'(x)$.

Start Tasks — Answers



19.

$$f'(x) = 14x^6 + 20x^3 - 6x$$

20.

$$f'(x) = 8x^7 - 10x^4 + 7$$

21.

$$f'(x) = 18x^5 - 12x^2 + 18x$$

22. $f'(x) = 2x^3 + \frac{3}{2}x - 1$

23.

$$f'(x) = \frac{16}{5}x^3 - \frac{12}{5}x^2 + 6x$$

24. $f'(x) = 2x^2 + x - 5$

25. $f'(x) = 2.5x^4 - 9.6x^3 - 10.5x^2 + 3.2x$

26. $f'(x) = 4.8x^3 + 2.4x^2 - 5x + 3.1$

27. $f'(x) = 18x^5 - 4.8x^3 + 8.4x^2 - 8.6x$

Build Tasks



1. $f(x) = (2x - 1)(x + 3)$.
First expand, then find $f'(x)$.

2.
 $f(x) = (x + 2)(3x - 5)$.
First expand, then find $f'(x)$.

3. $f(x) = x(x - 3)(x + 2)$.
First expand, then find $f'(x)$.

4. $f(x) = 2x^2(3x + 5)$.
First expand, then find $f'(x)$.

5.
 $f(x) = 4x^3(3x^2 - 2x + 1)$.
First expand, then find $f'(x)$.

6. $f(x) = (x + 1)(x^2 + 3)$.
First expand, then find $f'(x)$.

7.
 $f(x) = (3x - 2)(x^2 + 1)$.
First expand, then find $f'(x)$.

8. $f(x) = x^3(x - 4)$. First
expand, then find $f'(x)$.

9. $f(x) = x(x + 1)(x - 1)$.
First expand, then find $f'(x)$.

Build Tasks — Answers



1. $f'(x) = 4x + 5$

4. $f'(x) = 6x^2 + 10x$

7. $f'(x) = 9x^2 - 4x - 3$

2. $f'(x) = 6x - 1$

5. $f'(x) =$
 $48x^4 - 24x^3 + 12x^2$

8. $f'(x) = 4x^3 - 12x^2$

3. $f'(x) = 3x^2 - 2x - 6$

6. $f'(x) = 3x^2 + 2x + 3$

9. $f'(x) = 3x^2 - 1$

Build Tasks



10. $f(x) = \frac{4x^3 + 7x^2 - 6x - 5}{x^2}$.

Rewrite as separate powers of x , then differentiate.

13. $f(x) = \frac{3x^4}{2} + \frac{5}{x} + \frac{2}{x^2} + x$.

Rewrite as powers, then differentiate.

16. $f(x) = \frac{1}{x^2} + 2x^3 - \frac{3}{x}$.

Rewrite as powers, then differentiate.

11. $f(x) = 4x^7 + 2x - 3 + \frac{1}{x}$.

Rewrite $\frac{1}{x}$ as a power, then differentiate.

14. $f(x) = \frac{2}{x^3} + 3x^2 - 4x + \frac{1}{x}$.

Rewrite as powers, then differentiate.

17. $f(x) = \frac{2x^2 + 8x - 6}{4x}$.

Simplify first, then differentiate.

12. $f(x) = \frac{4x^3}{9} + \frac{2}{x} + x^2 + 1$.

Rewrite as powers, then differentiate.

15. $f(x) = x + \frac{1}{x} + \frac{1}{x^3}$.

Rewrite as powers, then differentiate.

18. $f(x) = \frac{3x^2 + 2x + x}{x}$.

Simplify first, then differentiate.

Build Tasks — Answers



10. $f'(x) =$
 $4x - 5x^{-2} + 7x^{-3} + 10x^{-4}$

13. $f'(x) =$
 $6x^3 - 5x^{-2} - 4x^{-3} + 1$

16. $f'(x) =$
 $-2x^{-3} + 6x^2 + 3x^{-2}$

11.
 $f'(x) = 28x^6 + 2 - x^{-2}$

14. $f'(x) =$
 $-6x^{-4} + 6x - 4 - x^{-2}$

17. $f'(x) = \frac{1}{2} + \frac{3}{2}x^{-2}$

12.
 $f'(x) = \frac{4}{3}x^2 - 2x^{-2} + 2x$

15.
 $f'(x) = 1 - x^{-2} - 3x^{-4}$

18. $f'(x) = 3$

Build Tasks



19. $f(x) = \frac{5x^4 - 8x^3 + 2x^2 + 3x}{x^3}$.

Simplify, then differentiate.

20. $f(x) = \frac{2x^3 + 6x^2 - 4x}{2x}$.

Simplify, then differentiate.

21. $f(x) = \frac{4x^5 - 2x^3 + 6x^2}{2x^2}$.

Simplify, then differentiate.

22. $f(x) = \frac{3x^4 - 9x^2 + 6x}{3x}$.

Simplify, then differentiate.

23. $f(x) = \frac{4x^6 - 8x^4 + 2x^2}{2x^2}$.

Simplify, then differentiate.

24. $f(x) = \frac{6x^3 - 3x^2 + 9x}{3x}$.

Simplify, then differentiate.

25. $f(x) = \frac{x^5 + 2x^3 - 4x^2}{x^2}$.

Simplify, then differentiate.

26. $f(x) = \frac{5x^5 - 10x^3 + 20x}{5x}$.

Simplify, then differentiate.

27. $f(x) = \frac{2x^4 - 6x^3 + 4x^2}{2x^2}$.

Simplify, then differentiate.

Build Tasks — Answers



19.

$$f'(x) = 10x - 8 + 3x^{-2}$$

22.

$$f'(x) = 3x^2 - 3 + 3x^{-1}$$

25. $f'(x) = 3x^2 + 2$

20. $f'(x) = 2x + 3$

23. $f'(x) = 6x^3 - 8x$

26. $f'(x) = 4x^2 - 3$

21. $f'(x) = 4x + \frac{3}{2}x^{-1}$

24. $f'(x) = 4x - 1$

27. $f'(x) = 4x - 3 + 2x^{-1}$

Push Tasks



1. $f(x) = \sqrt{x}$. Rewrite as a power, then differentiate.

2. $f(x) = \sqrt[3]{x}$. Rewrite as a power, then differentiate.

3. $f(x) = \sqrt{x^3}$. Rewrite as a power, then differentiate.

4. $f(x) = \frac{1}{x^2}$. Rewrite as a power, then differentiate.

5. $f(x) = \frac{1}{x^3}$. Rewrite as a power, then differentiate.

6. $f(x) = \frac{1}{x^4}$. Rewrite as a power, then differentiate.

7. $f(x) = \frac{5}{\sqrt[3]{x^2}}$. Rewrite as a power, then differentiate.

8. $f(x) = \frac{3}{\sqrt{x}}$. Rewrite as a power, then differentiate.

9. $f(x) = \frac{2}{\sqrt[4]{x^3}}$. Rewrite as a power, then differentiate.

Push Tasks — Answers



$$1. f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$4. f'(x) = -2x^{-3}$$

$$7. f'(x) = -\frac{10}{3}x^{-\frac{5}{3}}$$

$$2. f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

$$5. f'(x) = -3x^{-4}$$

$$8. f'(x) = -\frac{3}{2}x^{-\frac{3}{2}}$$

$$3. f'(x) = \frac{3}{2}x^{\frac{1}{2}}$$

$$6. f'(x) = -4x^{-5}$$

$$9. f'(x) = -\frac{3}{2}x^{-\frac{7}{4}}$$

Push Tasks



10.

$$f(x) = \frac{3}{\sqrt{x}} + \frac{2}{\sqrt[3]{x}} - \frac{9}{\sqrt[3]{x}}.$$

Simplify, then differentiate.

11.

$$f(x) = \frac{1}{x^3} - \frac{4}{x^4} + \frac{1}{\sqrt[3]{x^2}}.$$

Rewrite as powers, then differentiate.

12. $f(x) = \frac{3}{x} - \frac{2}{x^5} + \frac{5}{x^4}.$

Rewrite as powers, then differentiate.

13.

$$f(x) = \frac{4}{\sqrt{x}} - \frac{5}{x^2} + \frac{1}{\sqrt[3]{x}}.$$

Rewrite as powers, then differentiate.

14. $f(x) = \frac{2}{x^3} + \frac{3}{\sqrt{x}} - \frac{1}{x}.$

Rewrite as powers, then differentiate.

15.

$$f(x) = \frac{6}{\sqrt[3]{x^2}} - \frac{4}{x} + \frac{5}{\sqrt{x}}.$$

Rewrite as powers, then differentiate.

16.

$$f(x) = \frac{3}{x^2} + \frac{7}{\sqrt{x^3}} - 2x.$$

Rewrite as powers, then differentiate.

17.

$$f(x) = \frac{1}{\sqrt{x}} - \frac{3}{\sqrt[4]{x^3}} + \frac{2}{x^2}.$$

Rewrite as powers, then differentiate.

18.

$$f(x) = \frac{5}{x^4} - \frac{2}{\sqrt[3]{x}} + \frac{4}{x^2}.$$

Rewrite as powers, then differentiate.

Push Tasks — Answers



10.

$$f'(x) = -\frac{3}{2}x^{-\frac{3}{2}} - \frac{14}{3}x^{-\frac{4}{3}}$$

13. $f'(x) =$

$$-2x^{-\frac{3}{2}} + 10x^{-3} - \frac{1}{3}x^{-\frac{4}{3}}$$

16. $f'(x) =$

$$-6x^{-3} - \frac{21}{2}x^{-\frac{5}{2}} - 2$$

11. $f'(x) =$

$$-3x^{-4} + 16x^{-5} - \frac{2}{3}x^{-\frac{5}{3}}$$

14. $f'(x) =$

$$-6x^{-4} - \frac{3}{2}x^{-\frac{3}{2}} + x^{-2}$$

17. $f'(x) =$

$$-\frac{1}{2}x^{-\frac{3}{2}} + \frac{9}{4}x^{-\frac{7}{4}} - 4x^{-3}$$

12. $f'(x) =$

$$-3x^{-2} + 10x^{-6} - 20x^{-5}$$

15. $f'(x) =$

$$-4x^{-\frac{5}{3}} + 4x^{-2} - \frac{5}{2}x^{-\frac{3}{2}}$$

18. $f'(x) =$

$$-20x^{-5} + \frac{2}{3}x^{-\frac{4}{3}} - 8x^{-3}$$

Push Tasks



19. $f(x) = \frac{4x^3 + 7x^2 - 6x - 5}{x^2}$.

Simplify, then differentiate.

20. $f(x) = \frac{5x^4 - 8x^3 + 2x^2 + 3x}{x^3}$.

Simplify, then differentiate.

21. $f(x) = \frac{3x + \sqrt[3]{x^2}}{x}$.

Simplify, then differentiate.

22. $f(x) = \frac{2x^2 + \sqrt{x}}{x}$.

Simplify, then differentiate.

23.
 $f(x) = \frac{3x^4 - 2\sqrt{x^3} + x}{x^2}$.

Simplify, then differentiate.

24.
 $f(x) = \frac{5x^2 - \sqrt[3]{x} + 2x}{x}$.

Simplify, then differentiate.

25.
 $f(x) = \frac{4\sqrt{x} + 3x^2 - x}{x}$.

Simplify, then differentiate.

26. $f(x) = \frac{6x^3 - 4\sqrt[3]{x^2} + 2x}{2x}$.

Simplify, then differentiate.

27. $f(x) = \frac{2\sqrt{x^3} + 3x - 5\sqrt[4]{x}}{x}$.

Simplify, then differentiate.

Push Tasks — Answers



19.

$$f'(x) = 4 + 10x^{-3} + 12x^{-4}$$

20.

$$f'(x) = 10x - 8 + 3x^{-2}$$

21.

$$f'(x) = 3 - \frac{1}{3}x^{-\frac{4}{3}}$$

22. $f'(x) = 2 - \frac{1}{2}x^{-\frac{3}{2}}$

23.

$$f'(x) = 6x + 7x^{-\frac{3}{2}} - 2x^{-3}$$

24.

$$f'(x) = 5 + \frac{4}{3}x^{-\frac{4}{3}}$$

25. $f'(x) = -x^{-\frac{3}{2}} + 3$

26. $f'(x) = 6x + 2x^{-\frac{5}{3}}$

27. $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{5}{4}x^{-\frac{3}{4}}$