



Deriving functions

Mana Maths

Te reo Māori terms



pānga pārōnaki

derived function

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pānga pārōnaki

gradient function

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pārōnakitanga

differentiation

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mātāpono tuatahi

first principles

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Deriving from First Principles



Key idea



The derivative $f'(x)$ is the **gradient function**, found from first principles:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Method



1. Write $f(x+h)$.
2. Simplify $f(x+h) - f(x)$.
3. Divide by h .
4. Let $h \rightarrow 0$; the h terms vanish.

Common mistake

Expand in full: $(x+h)^2 = x^2 + 2xh + h^2$, **not** $x^2 + h^2$.



= **copy this into your book.** Boxes without a pencil are just to read.

First Principles - Worked Examples



Example: $f(x) = x^2$



$$f(x+h) = x^2 + 2xh + h^2$$

$$f(x+h) - f(x) = 2xh + h^2$$

$$\frac{2xh + h^2}{h} = 2x + h$$

$$\text{As } h \rightarrow 0: f'(x) = 2x$$

Example: $f(x) = 3x^2 + 2x$



$$f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$$

$$f(x+h) - f(x) = 6xh + 3h^2 + 2h$$

$$\frac{h(6x + 3h + 2)}{h} = 6x + 3h + 2$$

$$\text{As } h \rightarrow 0: f'(x) = 6x + 2$$

Common mistake

Keep every term with an h until the last step — only then let $h \rightarrow 0$.



= copy this into your book. Boxes without a pencil are just to read.

Start Tasks



1. Differentiate by first principles:

$$f(x) = x^2$$

2. Differentiate by first principles:

$$f(x) = 2x^2$$

3. Differentiate by first principles:

$$f(x) = 3x^2$$

4. Differentiate by first principles:

$$f(x) = x^2 + 5$$

5. Differentiate by first principles:

$$f(x) = x^2 - 7$$

6. Differentiate by first principles:

$$f(x) = 2x^2 + 3$$

7. Differentiate by first principles:

$$f(x) = 4x$$

8. Differentiate by first principles:

$$f(x) = 7x$$

9. Differentiate by first principles:

$$f(x) = -8x$$

Start Tasks - Answers



1. $f'(x) = 2x$

2. $f'(x) = 4x$

3. $f'(x) = 6x$

4. $f'(x) = 2x$

5. $f'(x) = 2x$

6. $f'(x) = 4x$

7. $f'(x) = 4$

8. $f'(x) = 7$

9. $f'(x) = -8$

Start Tasks



10. Differentiate by first principles:

$$f(x) = 4x + 5$$

11. Differentiate by first principles:

$$f(x) = 3x - 2$$

12. Differentiate by first principles:

$$f(x) = 5x + 1$$

13. Differentiate by first principles:

$$f(x) = x^2 + 4x$$

14. Differentiate by first principles:

$$f(x) = 2x^2 - 3x$$

15. Differentiate by first principles:

$$f(x) = x^2 + x$$

16. Differentiate by first principles:

$$f(x) = 3x^2 + 2x$$

17. Differentiate by first principles:

$$f(x) = -x^2 + 4x$$

18. Differentiate by first principles:

$$f(x) = 2x^2 + 5$$

Start Tasks - Answers



10. $f'(x) = 4$

11. $f'(x) = 3$

12. $f'(x) = 5$

13. $f'(x) = 2x + 4$

14. $f'(x) = 4x - 3$

15. $f'(x) = 2x + 1$

16. $f'(x) = 6x + 2$

17. $f'(x) = -2x + 4$

18. $f'(x) = 4x$

Start Tasks



19. Differentiate by first principles:

$$f(x) = x^2 + 2x + 1$$

20. Differentiate by first principles:

$$f(x) = 2x^2 - x + 3$$

21. Differentiate by first principles:

$$f(x) = x^2 + x - 4$$

22. Differentiate by first principles:

$$f(x) = 5x^2 - 2x + 1$$

23. Differentiate by first principles:

$$f(x) = -3x^2 + 4x - 2$$

24. Differentiate by first principles:

$$f(x) = 4x^2 + 3x - 5$$

25. Differentiate by first principles:

$$f(x) = x^2$$

26. Differentiate by first principles:

$$f(x) = 2x^2 - 3$$

27. Differentiate by first principles:

$$f(x) = 3x^2 - x$$

Start Tasks - Answers



19. $f'(x) = 2x + 2$

20. $f'(x) = 4x - 1$

21. $f'(x) = 2x + 1$

22. $f'(x) = 10x - 2$

23. $f'(x) = -6x + 4$

24. $f'(x) = 8x + 3$

25. $f'(x) = 2x$

26. $f'(x) = 4x$

27. $f'(x) = 6x - 1$

Build Tasks



1. Differentiate by first principles:

$$f(x) = 3x^2 + 2x + 4$$

2. Differentiate by first principles:

$$f(x) = x^2 + 5x$$

3. Differentiate by first principles:

$$f(x) = 5 - x^2$$

4. Differentiate by first principles:

$$f(x) = 2x^2 + 5x - 3$$

5. Differentiate by first principles:

$$f(x) = 4x^2 - 3x + 2$$

6. Differentiate by first principles:

$$f(x) = 3x^2 - 5x$$

7. Differentiate by first principles:

$$f(x) = -2x^2 + 4x - 1$$

8. Differentiate by first principles:

$$f(x) = 5x^2 + 3x$$

9. Differentiate by first principles:

$$f(x) = 6x^2 - 2x + 7$$

Build Tasks - Answers



1. $f'(x) = 6x + 2$

2. $f'(x) = 2x + 5$

3. $f'(x) = -2x$

4. $f'(x) = 4x + 5$

5. $f'(x) = 8x - 3$

6. $f'(x) = 6x - 5$

7. $f'(x) = -4x + 4$

8. $f'(x) = 10x + 3$

9. $f'(x) = 12x - 2$

Build Tasks



10. Differentiate by first principles:

$$f(x) = -4x^2 + 5x$$

11. Differentiate by first principles:

$$f(x) = 3x^2 + 7x - 2$$

12. Differentiate by first principles:

$$f(x) = 4 - 3x^2$$

13. Differentiate by first principles:

$$f(x) = 2x^2 - 4x + 5$$

14. Differentiate by first principles:

$$f(x) = -3x^2 - 2x + 6$$

15. Differentiate by first principles:

$$f(x) = 5x^2 - 3x + 4$$

16. Differentiate by first principles:

$$f(x) = 4x^2 + 2x - 3$$

17. Differentiate by first principles:

$$f(x) = 7x - 2x^2$$

18. Differentiate by first principles:

$$f(x) = 3x^2 - 4x + 1$$

Build Tasks - Answers



10. $f'(x) = -8x + 5$

11. $f'(x) = 6x + 7$

12. $f'(x) = -6x$

13. $f'(x) = 4x - 4$

14. $f'(x) = -6x - 2$

15. $f'(x) = 10x - 3$

16. $f'(x) = 8x + 2$

17. $f'(x) = 7 - 4x$

18. $f'(x) = 6x - 4$

Build Tasks



19. Differentiate by first principles:

$$f(x) = -x^2 + 6x - 5$$

20. Differentiate by first principles:

$$f(x) = 6x^2 + x - 4$$

21. Differentiate by first principles:

$$f(x) = 4x^2 - 5x + 3$$

22. Differentiate by first principles:

$$f(x) = 2x^2 + 3x - 7$$

23. Differentiate by first principles:

$$f(x) = 5 - 2x + 3x^2$$

24. Differentiate by first principles:

$$f(x) = -5x^2 + 2x + 3$$

25. Differentiate by first principles:

$$f(x) = 3 - 4x + x^2$$

26. Differentiate by first principles:

$$f(x) = 7x^2 - 5x + 2$$

27. Differentiate by first principles:

$$f(x) = 4x^2 + 5$$

Build Tasks - Answers



19. $f'(x) = -2x + 6$

20. $f'(x) = 12x + 1$

21. $f'(x) = 8x - 5$

22. $f'(x) = 4x + 3$

23. $f'(x) = 6x - 2$

24. $f'(x) = -10x + 2$

25. $f'(x) = 2x - 4$

26. $f'(x) = 14x - 5$

27. $f'(x) = 8x$

Push Tasks



1. Differentiate by first principles:

$$f(x) = x^3$$

2. Differentiate by first principles:

$$f(x) = x^3 + x$$

3. Differentiate by first principles:

$$f(x) = 2x^3$$

4. Differentiate by first principles:

$$f(x) = x^3 + x^2$$

5. Differentiate by first principles:

$$f(x) = 2x^3 - 3x$$

6. Differentiate by first principles:

$$f(x) = x^3 + 2x$$

7. Differentiate by first principles:

$$f(x) = 3x^3$$

8. Differentiate by first principles:

$$f(x) = x^3 - x^2 + x$$

9. Differentiate by first principles:

$$f(x) = 2x^3 + 4x$$

Push Tasks - Answers



1. $f'(x) = 3x^2$

2. $f'(x) = 3x^2 + 1$

3. $f'(x) = 6x^2$

4. $f'(x) = 3x^2 + 2x$

5. $f'(x) = 6x^2 - 3$

6. $f'(x) = 3x^2 + 2$

7. $f'(x) = 9x^2$

8. $f'(x) = 3x^2 - 2x + 1$

9. $f'(x) = 6x^2 + 4$

Push Tasks



10. Differentiate by first principles:
 $f(x) = ax^2 + bx$

11. Differentiate by first principles:
 $f(x) = x^3 + 2x^2$

12. Differentiate by first principles:
 $f(x) = 4x^3 - 5x$

13. Differentiate by first principles:
 $f(x) = ax^2$

14. Differentiate by first principles:
 $f(x) = x^3 - 3x^2$

15. Differentiate by first principles:
 $f(x) = 3x^3 + 2x^2$

16. Differentiate by first principles:
 $f(x) = ax + b$

17. Differentiate by first principles:
 $f(x) = 2x^3 + 3x^2 - x$

18. Differentiate by first principles:
 $f(x) = 4x^3 - 2x^2 + 3x$

Push Tasks - Answers



10. $f'(x) = 2ax + b$

11. $f'(x) = 3x^2 + 4x$

12. $f'(x) = 12x^2 - 5$

13. $f'(x) = 2ax$

14. $f'(x) = 3x^2 - 6x$

15. $f'(x) = 9x^2 + 4x$

16. $f'(x) = a$

17. $f'(x) = 6x^2 + 6x - 1$

18. $f'(x) = 12x^2 - 4x + 3$

Push Tasks



19. Differentiate by first principles:

$$f(x) = ax^2 + bx + c$$

20. Differentiate by first principles:

$$f(x) = 5x^3 - 2x$$

21. Differentiate by first principles:

$$f(x) = x^3 + 5x^2 + x$$

22. Differentiate by first principles:

$$f(x) = 2ax^2$$

23. Differentiate by first principles:

$$f(x) = 3x^3 - 4x^2 + 2x$$

24. Differentiate by first principles:

$$f(x) = ax^3$$

25. Differentiate by first principles:

$$f(x) = x^3 + 3x$$

26. Differentiate by first principles:

$$f(x) = 6x^3 - 2x^2$$

27. Differentiate by first principles:

$$f(x) = ax^2 + bx + c$$

(general)

Push Tasks - Answers



$$19. f'(x) = 2ax + b$$

$$20. f'(x) = 15x^2 - 2$$

$$21. f'(x) = 3x^2 + 10x + 1$$

$$22. f'(x) = 4ax$$

$$23. f'(x) = 9x^2 - 8x + 2$$

$$24. f'(x) = 3ax^2$$

$$25. f'(x) = 3x^2 + 3$$

$$26. f'(x) = 18x^2 - 4x$$

$$27. f'(x) = 2ax + b$$