



# Construction and Loci

Mana Maths

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# Te reo Māori terms



**kapehu**

compass

[Open in Te Aka](#)

**tahu**

line

[Open in Te Aka](#)

**porowhita**

circle

[Open in Te Aka](#)

**pewa**

arc

[Open in Te Aka](#)

# Loci - What They Are



## Key idea

A **locus** is the set of **all** points that follow a rule.

## The three main rules

One fixed point → **circle**.

Two points → **perpendicular bisector**.

Two lines → **angle bisector**.

## Example

Points exactly 4 cm from  $P$  form a circle, centre  $P$ , radius 4 cm.

## Common mistake

Do not give just one point. A locus is the **whole set** of points that fit the rule.

 = **copy this into your book.** Boxes without a pencil are just to read.

# Loci - Solving a Question



## Method

1. Turn each condition into a standard locus.
2. Draw each one carefully (keep the compass width fixed).
3. Where the loci **meet** are the answers.

## Example: two conditions

Same distance from A and B, **and** 5 cm from A:  
draw the perpendicular bisector of AB, then the circle centre A radius 5 cm.

## Common mistake

Watch the wording. 'Exactly 5 cm from P' is the circle; 'within 5 cm' shades the inside as well.

# Start Tasks



**1.** Which tool is best for drawing a circle with a fixed radius: ruler or compass?

**2.** Which tool is best for joining two marked points with a straight segment?

**3.** What is the locus of points exactly 3 cm from point  $P$ ?

**4.** What is the locus of points that are the same distance from points  $A$  and  $B$ ?

**5.** What is the locus of points that are the same distance from the two arms of an angle?

**6.** You draw a circle with centre  $O$  and radius 4 cm. How far is every point on the circle from  $O$ ?

**7.** If point  $C$  lies on the perpendicular bisector of  $AB$ , what can you say about  $CA$  and  $CB$ ?

**8.** What is the locus of points exactly 2 cm from line  $l$ ?

**9.** What does the word *equidistant* mean?

# Start Tasks – Answers



1. Which tool is best for drawing a circle with a fixed radius: ruler or compass?

**Solution** Compass — it holds a fixed radius.

2. Which tool is best for joining two marked points with a straight segment?

**Solution** Ruler — it draws a straight line.

3. What is the locus of points exactly 3 cm from point  $P$ ?



**Solution** A circle, centre  $P$ , radius 3 cm.

4. What is the locus of points that are the same distance from points  $A$  and  $B$ ?

**Solution** The perpendicular bisector of  $AB$ .

5. What is the locus of points that are the same distance from the two arms of an angle?

**Solution** The bisector of the angle.

6. You draw a circle with centre  $O$  and radius 4 cm. How far is every point on the circle from  $O$ ?

**Solution** 4 cm — every point is one radius from the centre.

7. If point  $C$  lies on the perpendicular bisector of  $AB$ , what can you say about  $CA$  and  $CB$ ?

**Solution**  $CA = CB$  — the point is equidistant from  $A$  and  $B$ .

8. What is the locus of points exactly 2 cm from line  $l$ ?



**Solution** Two parallel lines, one 2 cm on each side of  $l$ .

9. What does the word *equidistant* mean?

**Solution** The same distance from each.

# Start Tasks



**10.** A park must be built the same distance from two roads that cross. Which construction line shows all possible places?

**13.** When constructing the perpendicular bisector of  $AB$ , why do you use the same compass width from  $A$  and from  $B$ ?

**16.** How many compass arcs are usually drawn first when making a perpendicular bisector of  $AB$ ?

**11.** If a point lies on a circle with centre  $P$  and radius 6 cm, what is its distance from  $P$ ?

**14.** Which locus can have two separate branches: points 2 cm from a line, or points 2 cm from a point?

**17.** A point lies on the bisector of an  $80^\circ$  angle. What two equal angles are formed?

**12.** The midpoint of segment  $AB$  always lies on which special line of  $AB$ ?

**15.** What region is described by points less than 3 cm from point  $P$ ?

**18.** Which circle is farther from the centre: radius 3 cm or radius 5 cm?

# Start Tasks – Answers



**10.** A park must be equal distance from two crossing roads. Which construction line shows all possible places?

**Solution** The angle bisector of the roads.

**11.** If a point lies on a circle with centre  $P$  and radius 6 cm, what is its distance from  $P$ ?

**Solution** 6 cm.

**12.** The midpoint of segment  $AB$  always lies on which special line of  $AB$ ?

**Solution** The perpendicular bisector of  $AB$ .

**13.** When constructing the perpendicular bisector of  $AB$ , why use the same compass width from both points?

**Solution** So the arc crossings are equally far from  $A$  and  $B$ .

**14.** Which locus can have two separate branches: points 2 cm from a line, or points 2 cm from a point?

**Solution** Points 2 cm from a line — one branch each side.

**15.** What region is described by points less than 3 cm from point  $P$ ?

**Solution** Inside the circle of radius 3 cm about  $P$  (boundary excluded).

**16.** How many compass arcs are usually drawn first when making a perpendicular bisector of  $AB$ ?

**Solution** Two — one from each end.

**17.** A point lies on the bisector of an  $80^\circ$  angle. What two equal angles are formed?

**Solution**  $40^\circ$  and  $40^\circ$  ( $80^\circ \div 2$ ).

**18.** Which circle is farther from the centre: radius 3 cm or radius 5 cm?

**Solution** The radius 5 cm circle.

# Start Tasks



**19.** If you only want the points 4 cm from  $Q$  on one side of  $Q$ , would you draw a full circle or just an arc?

**20.** Which construction finds points equally far from  $A$  and  $B$ ?

**21.** Which construction finds points equally far from two intersecting lines?

**22.** A school must be within 2 km of the town hall. Should you shade inside or outside the circle locus?

**23.** Why must the compass width stay fixed while one circle is being drawn?

**24.** If two equal circles centred at  $A$  and  $B$  cross, how many intersection points do they usually have?

**25.** Which locus is closed: a circle or a perpendicular bisector?

**26.** Can the midpoint of  $AB$  be found where the perpendicular bisector crosses  $AB$ ?

**27.** If a point is on the perpendicular bisector of  $AB$ , is it closer to  $A$ , closer to  $B$ , or the same distance from both?

# Start Tasks – Answers



**19.** If you only want the points 4 cm from Q on one side of Q, would you draw a full circle or just an arc?

**Solution** Just an arc — a full circle covers both sides.

**20.** Which construction finds points equally far from A and B?



**Solution** The perpendicular bisector of AB.

**21.** Which construction finds points equally far from two intersecting lines?



**Solution** The angle bisector.

**22.** A school must be within 2 km of the town hall. Should you shade inside or outside the circle locus?

**Solution** Inside (and on) the circle of radius 2 km.

**23.** Why must the compass width stay fixed while one circle is being drawn?

**Solution** To keep the radius constant so the curve is a true circle.

**24.** If two equal circles centred at A and B cross, how many intersection points do they usually have?

**Solution** Two.

**25.** Which locus is closed: a circle or a perpendicular bisector?

**Solution** The circle — it closes on itself; the bisector runs on forever.

**26.** Can the midpoint of AB be found where the perpendicular bisector crosses AB?

**Solution** Yes — the bisector meets AB at its midpoint.

**27.** A point on AB's perpendicular bisector: closer to A, closer to B, or equidistant?

**Solution** Equidistant from A and B.

# Build Tasks



**1.** A school must be built the same distance from towns  $A$  and  $B$ . On which line should the school lie?

**2.** A dog is tied to peg  $P$  with a 3 m lead. What locus shows all possible positions of the dog?

**3.** A path must stay exactly 2 m from a straight fence. What locus should be drawn?

**4.** Two roads meet at a corner. A bus stop must be the same distance from both roads. Which line gives its possible positions?

**5.** If point  $C$  is on the perpendicular bisector of  $AB$  and  $CA = 7$  cm, find  $CB$ .

**6.** A point lies on the bisector of an  $80^\circ$  angle. What are the two equal angles?

**7.** A playground must be within 5 m of tree  $T$ . Describe the region that should be shaded.

**8.** A lamp post must be the same distance from the two walls of a corner. Which locus should be used?

**9.** Two circles of radius 4 cm are centred at  $A$  and  $B$ . If they meet at  $X$ , what are  $XA$  and  $XB$ ?

# Build Tasks – Answers



1. A school must be built the same distance from towns A and B. On which line should the school lie?

**Solution** The perpendicular bisector of AB.

2. A dog is tied to peg P with a 3 m lead. What locus shows all possible positions of the dog?

**Solution** A circle, centre P, radius 3 m.

3. A path must stay exactly 2 m from a straight fence. What locus should be drawn?

**Solution** Two parallel lines, 2 m each side of the fence.

4. Two roads meet at a corner. A bus stop must be equal distance from both. Which line gives its positions?

**Solution** The angle bisector of the two roads.

5. If point C is on the perpendicular bisector of AB and  $CA = 7$  cm, find CB.



**Solution**  $CB = 7$  cm — on the bisector  $CA = CB$ .

6. A point lies on the bisector of an  $80^\circ$  angle. What are the two equal angles?

**Solution**  $40^\circ$  and  $40^\circ$  ( $80^\circ \div 2$ ).

7. A playground must be within 5 m of tree T. Describe the region that should be shaded.

**Solution** The disc inside and on the circle of radius 5 m about T.

8. A lamp post must be equal distance from the two walls of a corner. Which locus is used?

**Solution** The angle bisector of the corner.

9. Two circles of radius 4 cm are centred at A and B. If they meet at X, what are XA and XB?

**Solution**  $XA = XB = 4$  cm — X lies on each circle.

# Build Tasks



**10.** To find the midpoint of ribbon  $AB$ , which special line do you construct first?

**11.** What is the locus of points exactly 3 cm from line  $l$ ?

**12.** A rescue point must be equal distance from wharves  $A$  and  $B$ , and also 2 km from lighthouse  $L$ . Which two loci should be

**13.** Why must the compass radius be more than half of  $AB$  when constructing the perpendicular bisector of  $AB$ ?

**14.** Two straight paths cross. How many angle bisector lines are possible?

**15.** Which set is larger: points exactly 4 cm from  $P$ , or points at most 4 cm from  $P$ ?

**16.** A circle has centre  $O$  and radius 6 cm. Is a point 8 cm from  $O$  on the locus?

**17.** If two loci do not cross at all, how many solutions does the problem have?

**18.** If two loci just touch once, how many solutions are there?

# Build Tasks – Answers



**10.** To find the midpoint of ribbon  $AB$ , which special line do you construct first?

**Solution** The perpendicular bisector of  $AB$ .

**11.** What is the locus of points exactly 3 cm from line  $l$ ?



**Solution** Two parallel lines, 3 cm each side of  $l$ .

**12.** A rescue point is equidistant from wharves  $A$  and  $B$ , and 2 km from lighthouse  $L$ . Which two loci intersect?

**Solution** The perpendicular bisector of  $AB$  and the circle, centre  $L$ , radius 2 km.

**13.** Why must the compass radius exceed half of  $AB$  when constructing its perpendicular bisector?

**Solution** Otherwise the two arcs are too short to cross.

**14.** Two straight paths cross. How many angle bisector lines are possible?



**Solution** Two — the internal and external bisectors (at right

**15.** Which set is larger: points exactly 4 cm from  $P$ , or points at most 4 cm from  $P$ ?

**Solution** At most 4 cm — the whole disc, not just the boundary.

**16.** A circle has centre  $O$  and radius 6 cm. Is a point 8 cm from  $O$  on the locus?

**Solution** No —  $8\text{ cm} \neq 6\text{ cm}$ .

**17.** If two loci do not cross at all, how many solutions does the problem have?

**Solution** Zero.

**18.** If two loci just touch once, how many solutions are there?

**Solution** One.

# Build Tasks



**19.** If two loci cross twice, how many solutions are there?

**20.** Describe the region inside radius 3 cm of  $O$  but outside radius 1 cm of the same centre  $O$ .

**21.** A cyclist must stay 1 m from a painted centre line on either side. What locus shows all allowed wheel tracks?

**22.** Which tool should be used to draw a circle accurately: compass or ruler?

**23.** What measurement stays fixed while a circle is being constructed?

**24.** If  $X$  and  $Y$  are the intersections of equal-radius arcs from  $A$  and  $B$ , what line passes through  $X$  and  $Y$ ?

**25.** If a  $70^\circ$  angle is bisected, what two equal angles are formed?

**26.** What is the locus of points equal distance from two parallel lines?

**27.** A place must be equal distance from  $A$  and  $B$  and also north of segment  $AB$ . Which part of the perpendicular

# Build Tasks – Answers



**19.** If two loci cross twice, how many solutions are there?

**Solution** Two.

**20.** Describe the region inside radius 3 cm of  $O$  but outside radius 1 cm of the same centre  $O$ .

**Solution** The ring (annulus) between the 1 cm and 3 cm circles.

**21.** A cyclist must stay 1 m from a painted centre line on either side. What locus shows all allowed wheel tracks?

**Solution** Two parallel lines, 1 m each side of the centre line.

**22.** Which tool should be used to draw a circle accurately: compass or ruler?

**Solution** A compass.

**23.** What measurement stays fixed while a circle is being constructed?

**Solution** The radius.

**24.** If  $X$  and  $Y$  are the intersections of equal-radius arcs from  $A$  and  $B$ , what line passes through  $X$  and  $Y$ ?

**Solution** The perpendicular bisector of  $AB$ .

**25.** If a  $70^\circ$  angle is bisected, what two equal angles are formed?

**Solution**  $35^\circ$  and  $35^\circ$  ( $70^\circ \div 2$ ).

**26.** What is the locus of points equal distance from two parallel lines?

**Solution** The line halfway between them, parallel to both.

**27.** A place is equidistant from  $A$ ,  $B$ , and north of  $AB$ . Which bisector part is relevant?

**Solution** The part of the perpendicular bisector of  $AB$  north of  $AB$ .

# Push Tasks



**1.** Points must be equal distance from  $A$  and  $B$ , and also 5 cm from  $A$ . What two loci should be intersected?

**2.** If  $AB = 12$  cm, can a point be equal distance from  $A$  and  $B$  and also only 4 cm from  $A$ ? Explain briefly.

**3.** Two roads cross at  $60^\circ$ . After both angle bisectors are drawn, what angle sizes appear between the roads and the bisectors?

**4.** Explain why every point on the perpendicular bisector of  $AB$  is the same distance from  $A$  and  $B$ .

**5.** A campsite must be exactly 4 km from lake  $L$  and the same distance from towns  $A$  and  $B$ . How many possible sites can there be at most?

**6.** A sprinkler reaches exactly 6 m from point  $P$ , but only south of  $P$ . What part of the circle locus is needed?

**7.** Two parallel fences are 10 m apart. What is the locus of points equal distance from both fences?

**8.** Why does changing the compass width while drawing a circle break the locus condition?

**9.** Why can a ruler-without-markings and a compass still find the midpoint of  $AB$ ?

# Push Tasks – Answers



1. Points must be equal distance from  $A$  and  $B$ , and also 5 cm from  $A$ . What two loci should be intersected?

**Solution** The perpendicular bisector of  $AB$  and the circle, centre  $A$ , radius 5 cm.

4. Explain why every point on the perpendicular bisector of  $AB$  is the same distance from  $A$  and  $B$ .

**Solution** Reflecting in the bisector swaps  $A$  and  $B$ , so each of its points is equidistant.

7. Two parallel fences are 10 m apart. What is the locus of points equal distance from both?

**Solution** The line midway between them (5 m from each),

2. If  $AB = 12$  cm, can a point be equal distance from  $A$  and  $B$  and also only 4 cm from  $A$ ? Explain briefly.

**Solution** No — the bisector's nearest point to  $A$  is the midpoint, 6 cm away.

5. A campsite is exactly 4 km from lake  $L$  and equal distance from towns  $A$  and  $B$ . At most, how many sites are possible?

**Solution** At most two — a line meets a circle in up to 2 points.

8. Why does changing the compass width while drawing a circle break the locus condition?

**Solution** The distance from the centre stops being constant, so

3. Two roads cross at  $60^\circ$ . Once both angle bisectors are drawn, what angles do they make with the roads?

**Solution** Internal bisector  $30^\circ$ ; external bisector  $60^\circ$ .

6. A sprinkler reaches exactly 6 m from point  $P$ , but only south of  $P$ . What part of the circle locus is needed?

**Solution** The southern half — a semicircular arc.

9. Why can a ruler-without-markings and a compass still find the midpoint of  $AB$ ?

**Solution** Equal arcs from  $A$  and  $B$  give equidistant points; their

# Push Tasks



**10.** A point lies on the internal bisector of an  $80^\circ$  angle and another lies on the external bisector. What angle lies between the two

**13.** A path must stay within 2 m of line  $l$ . Is the boundary a pair of lines or a circle?

**16.** If the perpendicular bisector and a circle do not meet, what does that tell you about the construction problem?

**11.** A school must be equal distance from  $A$  and  $B$  and inside a 3 km safety circle around  $C$ . Describe the final solution set

**14.** A point is on the perpendicular bisector of  $AB$  and  $AX = 9$  cm. What is  $BX$ ?

**17.** If they meet in two places, why are there two valid answers?

**12.** Two equal circles have centres  $A$  and  $B$ , and  $AB$  equals the radius. What triangle is formed by  $A$ ,  $B$ , and one intersection point?

**15.** A point is on an angle bisector of  $34^\circ$ . What are the two equal angles?

**18.** A builder wants a point exactly 3 m from wall  $w$  and exactly 3 m from wall  $v$ , where the walls cross. Which locus

# Push Tasks – Answers



**10.** One point is on the internal, another on the external bisector of an  $80^\circ$  angle. Angle between them?

**Solution**  $90^\circ$  — the two bisectors are always perpendicular.

**13.** A path must stay within 2 m of line  $l$ . Is the boundary a pair of lines or a circle?



**Solution** A pair of parallel lines, 2 m each side of  $l$ .

**16.** A perpendicular bisector and a circle do not meet. What does that tell you?

**Solution** No point meets both conditions — the problem has no solution.

**11.** A school is equal distance from A and B, inside a 3 km safety circle around C. Describe the solution set.

**Solution** The part of the perpendicular bisector of AB inside the 3 km circle about C.

**14.** A point is on the perpendicular bisector of AB and  $AX = 9$  cm. What is BX?



**Solution**  $BX = 9$  cm — on the bisector  $AX = BX$ .

**17.** If they meet in two places, why are there two valid answers?

**Solution** Each crossing point satisfies both loci, so both are valid.

**12.** Two equal circles centred at A, B have AB equal to the radius. What triangle do A, B, and an intersection form?

**Solution** An equilateral triangle — all three sides equal the radius.

**15.** A point is on an angle bisector of  $34^\circ$ . What are the two equal angles?



**Solution**  $17^\circ$  and  $17^\circ$  ( $34^\circ \div 2$ ).

**18.** A point must be 3 m from crossing walls w, v. Better to start with: circle or angle bisector?

**Solution** The angle bisector (equal distance from both

# Push Tasks



**19.** A point must be equal distance from two parallel lines and also 4 cm from point  $P$  on the midline. How many

solutions can there be at

**22.** What is the locus of points exactly halfway between two parallel roads?

**25.** A point must be exactly 2 cm from line  $l$  and exactly 2 cm from point  $P$ . What usually finds the answer: one

**20.** Why is the midpoint of  $AB$  guaranteed to lie on the perpendicular bisector of  $AB$ ?

**23.** Give one reason why an angle bisector is useful in real design or surveying work.

**26.** Why do construction problems often end by checking where two loci cross?

**21.** A circle has centre  $O$ . Is the centre itself on the locus of points 5 cm from  $O$ ? Explain.

**24.** If a point is inside a circle of radius 5 cm, can it be on the locus of points exactly 5 cm from the centre?

**27.** Write one sentence that explains what a locus is.

# Push Tasks – Answers



**19.** A point is equidistant from two parallel lines, and 4 cm from  $P$  on the midline. Max solutions?

**Solution** At most two — the midline meets the circle in up to 2 points.

**22.** What is the locus of points exactly halfway between two parallel roads?

**Solution** The middle line, parallel to both roads.

**25.** A point is exactly 2 cm from line  $l$  and from  $P$ . One locus, or the intersection of two?

**Solution** The intersection of two loci — the lines 2 cm from  $l$  and the circle 2 cm from  $P$ .

**20.** Why is the midpoint of  $AB$  guaranteed to lie on the perpendicular bisector of  $AB$ ?

**Solution** It is equally far from  $A$  and  $B$ , which is exactly the bisector's rule.

**23.** Give one reason why an angle bisector is useful in real design or surveying work.

**Solution** It centres a path or feature equally between two edges or boundaries.

**26.** Why do construction problems often end by checking where two loci cross?

**Solution** Each crossing point satisfies every condition at once, giving the solution.

**21.** A circle has centre  $O$ . Is the centre itself on the locus of points 5 cm from  $O$ ? Explain.

**Solution** No —  $O$  is 0 cm from itself, not 5 cm.

**24.** If a point is inside a circle of radius 5 cm, can it be on the locus of points exactly 5 cm from the centre?

**Solution** No — inside points are under 5 cm; the locus is the boundary only.

**27.** Write one sentence that explains what a locus is.

**Solution** A locus is the set of all points that obey a given rule.