Conditional and independent events

Mana Maths

Te reo Māori terms

tūponotanga
probability

Open in Te Aka

takahanga
event

Open in Te Aka

herenga
conditional

Open in Te Aka

motuhake
independent

Open in Te Aka

Conditional and independent events

Independent and Dependent Events

Key idea

Events are **independent** if knowing one happened does not change the probability of the other. If it does, the events are **dependent**.

Dependent example

Bag: 3 red, 2 blue. Draw red, keep it out. Now 2 red, 2 blue — the next probability changed.

Independent example

If the counter is put back, the bag returns to 3 red, 2 blue — the next probability is unchanged.

Common mistake

Independent does **not** mean 'separate' or 'different'. Two events can happen apart in time and still be dependent.

= copy this into your book Boxes without a pencil are just to read

Conditional and independent events

Conditional Probability

Key idea

'Given' means we already know some information. **Conditional probability** is the probability worked out inside that smaller, known group.

Example

The probability a chosen red card is a heart looks only inside the 26 red cards, not all 52.

Clue words

Without replacement usually means **dependent**. With replacement usually means **independent**.

Common mistake

Fewer outcomes does not always mean a smaller probability — it can go up, down, or stay the same.

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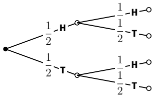
Start Tasks

Decide whether two events are independent or dependent.

1.



Two fair coins tossed. Does the 1st affect the 2nd? Independent?

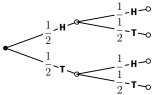
2. A card is drawn and NOT replaced. Is the 2nd draw independent of the 1st? Explain.

3. What does “independent events” mean, in your own words?

4. What does “conditional probability” mean, in your own words?

Start Tasks — Answers

1. Two fair coins tossed. Does the 1st affect the 2nd? Independent?



Solution

Yes – independent.

2. A card is drawn and NOT replaced. Is the 2nd draw independent of the 1st? Explain.

Solution

No – removing the card changes what is left, so the 2nd draw is affected.

3. What does “independent events” mean, in your own words?

Solution

One event happening does not change
the probability of the other.

4. What does “conditional probability” mean, in your own words?

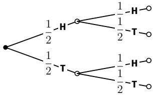
Solution

The probability of an event given that another event has already happened.

Start Tasks

Use the multiplication rule for independent events, and spot the role of replacement.

5.



For two fair coin tosses, find $P(\text{Heads then Heads})$.

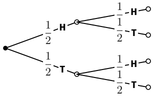
6. A marble is drawn, then replaced, then a second is drawn. Are the draws independent? Why?

7. A die is rolled twice. Are the two rolls independent?

8. Why does “replacement” usually make two draws independent?

Start Tasks — Answers

5. For two fair coin tosses, find $P(\text{Heads then Heads})$.



Solution

$$P(H,H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

6. A marble is drawn, then replaced, then a second is drawn. Are the draws independent? Why?

Solution

Yes – replacing the marble restores the original probabilities.

7. A die is rolled twice. Are the two rolls independent?

Solution

Yes – each roll starts fresh; the die has no memory.

8. Why does "replacement" usually make two draws independent?

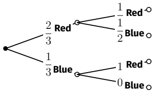
Solution

Replacement restores the original set, so the 2nd draw's chances are unchanged.

Build Tasks

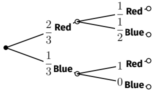
Use a without-replacement tree to find conditional probabilities, $P(B \mid A)$.

1.



Bag: 2 red, 1 blue (no replacement). Find $P(\text{2nd Red} \mid \text{1st Red})$.

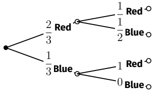
2.



Using the same tree, find $P(\text{2nd is Red} \mid \text{1st was Blue})$.

3. What does $P(B \mid A)$ mean, in words?

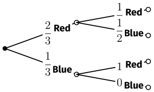
4.



Using the tree, find $P(\text{Red then Red})$ by multiplying along the branches.

Build Tasks — Answers

1. Bag: 2 red, 1 blue (no replacement). Find $P(\text{2nd Red} \mid \text{1st Red})$.

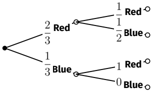


Solution

Branch after Red:

$P = \frac{1}{2}$

2. Using the same tree, find $P(\text{2nd is Red} \mid \text{1st was Blue})$.



Solution

Branch after Blue:

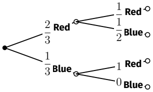
$P = 1$

3. What does $P(B \mid A)$ mean, in words?

Solution

The probability that B happens, given that A has already happened.

4. Using the tree, find $P(\text{Red then Red})$ by multiplying along the branches.



Solution

$P(R, R) = \frac{2}{3} \times \frac{1}{2}$

$= \frac{1}{3}$

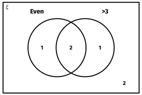
Build Tasks

Test independence by comparing $P(B)$ with $P(B \mid A)$.

5. $P(B) = 0.4$ and $P(B \mid A) = 0.4$. Are A and B independent?

6. $P(B) = 0.4$ but $P(B \mid A) = 0.7$. Are A and B independent? Explain.

7.



Die: $A =$even, $B => 3$. Using the overlap, independent?

8. Why does comparing $P(B)$ to $P(B \mid A)$ test for independence?

Build Tasks — Answers

5. $P(B) = 0.4$ and $P(B \mid A) = 0.4$. Are A and B independent?

Solution

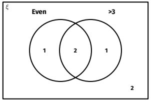
Yes – the condition does not change $P(B)$, so A and B are independent.

6. $P(B) = 0.4$ but $P(B \mid A) = 0.7$. Are A and B independent? Explain.

Solution

No – $P(B \mid A) \neq P(B)$, so A changes the chance of B: dependent.

7. Die: $A =$even, $B => 3$. Using the overlap, independent?



Solution

$P(A) = \frac{1}{2}$ vs $\frac{2}{3}$: dependent.

8. Why does comparing $P(B)$ to $P(B \mid A)$ test for independence?

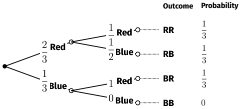
Solution

If they are equal, A has no effect on B; if different, B depends on A.

Push Tasks

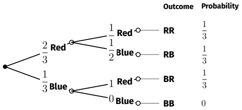
Calculate combined probabilities from a without-replacement tree.

1.



Find $P(\text{one Red, one Blue, either order})$.

2.



Find $P(\text{at least one Blue})$.

3. A student says "A card is shuffled and drawn, so every later draw must be independent of it." What is wrong with this?

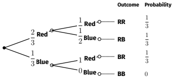
4.

Hand/Guitar	Guitar	No guitar	Total
Left-handed	$\boxed{6}$	14	20
Right-handed	12	68	80
Total	18	82	100

Find $P(\text{Guitar} \mid \text{Left-handed})$ vs $P(\text{Guitar})$ overall. Independent?

Push Tasks — Answers

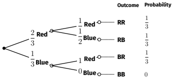
1. Find $P(\text{one Red, one Blue, either order})$.



Solution

$P(RB) + P(BR) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

2. Find $P(\text{at least one Blue})$.



Solution

$1 - P(RR) = 1 - \frac{1}{3} = \frac{2}{3}$

3. A student says "A card is shuffled and drawn, so every later draw must be independent of it." What is wrong with this?

Solution

Later draws aren't independent.

4. Find $P(\text{Guitar} \mid \text{Left-handed})$ vs $P(\text{Guitar})$ overall. Independent?

Hand/Guitar	Guitar	No guitar	Total
Left-handed	6	14	20
Right-handed	12	68	80
Total	18	82	100

Solution

$P(G \mid L) = 0.3$, $P(G) = 0.18$: dependent.

Push Tasks

Critique probability claims involving conditional and independent reasoning.

5. "I already rolled a 6, so the next roll is less likely to be a 6." Explain the error.

6. A survey shows many left-handed students play guitar. Does that alone prove dependence? What else matters?

7. If event A makes B more likely, what can you say about $P(B \mid A)$ compared with $P(B)$?

8. Describe one real example of independent events and one of dependent (conditional) events.

Push Tasks — Answers

5. "I already rolled a 6, so the next roll is less likely to be a 6." Explain the error.

Solution

Gambler's fallacy: $P(6) = \frac{1}{6}$ always.

6. A survey shows many left-handed students play guitar. Does that alone prove dependence? What else matters?

Solution

No – check $P(G \mid L)$ too.

7. If event A makes B more likely, what can you say about $P(B \mid A)$ compared with $P(B)$?

Solution

$P(B \mid A) > P(B)$.

8. Describe one real example of independent events and one of dependent (conditional) events.

Solution

Coin tosses; cards w/o replacement.