Circle theorem

Mana Maths

Te reo Māori terms

porowhita
circle
Open in Te Aka

putoro
radius
Open in Te Aka

patapa
tangent
Open in Te Aka

porowhita haurua
semi-circle
Open in Te Aka

Circle theorem

Centre and Circumference Angles

Key idea

The **centre angle** is double the **circumference angle** on the same arc.

An angle in a **semicircle** is always $90°$.

Example

Centre angle $= 96°$

$\Rightarrow$ circumference angle $= 48°$

Same segment

Angles on the same arc, standing in the same segment, are always **equal** to each other.

Common mistake

Same-segment angles are **equal**, not double. Only the centre angle is double the circumference angle.

= copy this into your book. ~~Boxes without a pencil are just to read.~~

Circle theorem

Tangents and Cyclic Quadrilaterals

Key idea

A **tangent** meets the radius at $90°$ at the point of contact.

Opposite angles in a **cyclic quadrilateral** sum to $180°$.

Example

One cyclic quadrilateral angle $= 113°$

$\Rightarrow$ opposite angle $= 180° - 113° = 67°$

Order of working

Spot the theorem the diagram shows, write it as an equation, then solve.

Common mistake

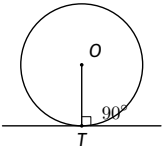
Don't confuse 'equal' with 'supplementary': same-segment angles are equal; cyclic-quad opposite angles add to $180°$.

= copy this into your book — Boxes without a pencil are just to read

Start Tasks

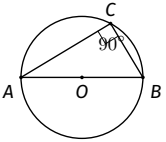
State the tangent-radius and angle-in-a-semicircle theorems.

1.



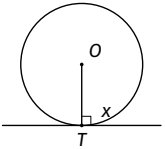
What is the tangent-radius angle?

2.



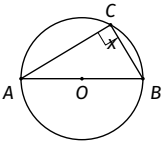
What is the angle in a semicircle?

3.



Find x.

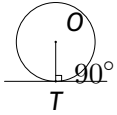
4.



Find x.

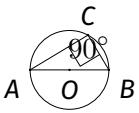
Start Tasks — Answers

1. What is the tangent-radius angle?



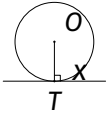
Solution
A tangent always meets the radius at $90°$.

2. What is the angle in a semicircle?



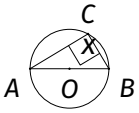
Solution
The angle in a semicircle is always $90°$.

3. Find x.



Solution
Tangent–radius angle $= 90°$
$x = 90°$

4. Find x.

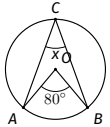


Solution
Angle in a semicircle $= 90°$
$x = 90°$

Start Tasks

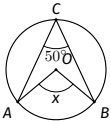
Use the centre/circumference theorem to find a missing angle.

5.



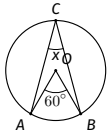
Find x.

6.



Find x.

7.

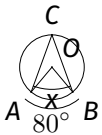


Find x.

8. The angle at the centre of a circle is $120°$. Find the angle at the circumference on the same arc.

Start Tasks — Answers

5. Find x.



Solution

Centre $= 2\times$ circumference

$x = 80 \div 2 = 40°$

6. Find x.

Solution

Centre $= 2\times$ circumference

$x = 2 \times 50 = 100°$

7. Find x.

Solution

Centre $= 2\times$ circumference

$x = 60 \div 2 = 30°$

8. The angle at the centre of a circle is $120°$. Find the angle at the circumference on the same arc.

Solution

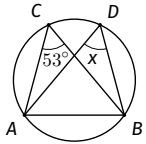
Circumference $=$ centre $\div 2$

$= 120 \div 2 = 60°$

Start Tasks

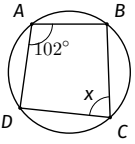
Use the same-segment and cyclic-quadrilateral theorems to find a missing angle.

9.



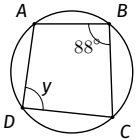
Find x.

10.



Find x.

11.



Find y.

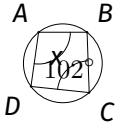
12. Opposite angles in a cyclic quadrilateral are $x°$ and $70°$. If they sum to $180°$, find x.

Start Tasks — Answers

9. Find x.

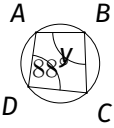
Solution
Angles in the same segment are equal: $x = 53°$

10. Find x.



Solution
Opposite angles sum to $180°$
$x = 180 - 102 = 78°$

11. Find y.



Solution
Opposite angles sum to $180°$
$y = 180 - 88 = 92°$

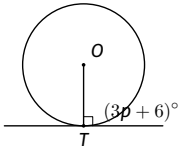
12. Opposite angles in a cyclic quadrilateral are $x°$ and $70°$. If they sum to $180°$, find x.

Solution
$x + 70 = 180$
$x = 110°$

Build Tasks

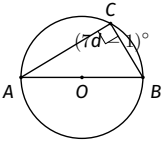
Form and solve an equation from the tangent-radius or semicircle theorem.

1.



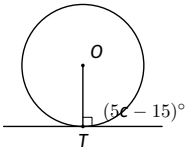
Find p.

2.



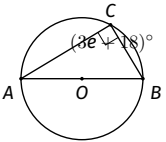
Find d.

3.



Find c.

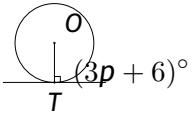
4.



Find e.

Build Tasks — Answers

1. Find p.

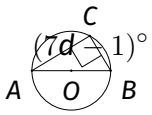


Solution

$3p + 6 = 90$

$3p = 84 \Rightarrow p = 28$

2. Find d.

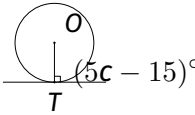


Solution

$7d - 1 = 90$

$7d = 91 \Rightarrow d = 13$

3. Find c.

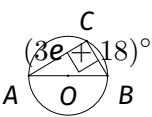


Solution

$5c - 15 = 90$

$5c = 105 \Rightarrow c = 21$

4. Find e.



Solution

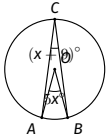
$3e + 18 = 90$

$3e = 72 \Rightarrow e = 24$

Build Tasks

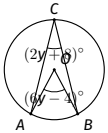
Form and solve an equation from the centre/circumference theorem.

5.



Find x.

6.



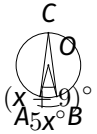
Find y.

7. The centre angle is $(9m+3)^\circ$. The same-arc circumference angle is $(3m+9)^\circ$. Find m.

8. The centre angle is 140°. The circumference angle is $(4n-10)^\circ$. Find n.

Build Tasks — Answers

5. Find x.



Solution

$5x = 2(x + 9)$

$3x = 18 \Rightarrow x = 6$

6. Find y.

Solution

$6y - 4 = 2(2y + 8)$

$2y = 20 \Rightarrow y = 10$

7. The centre angle is $(9m + 3)°$. The same-arc circumference angle is $(3m + 9)°$. Find m.

Solution

$9m + 3 = 2(3m + 9)$

$3m = 15 \Rightarrow m = 5$

8. The centre angle is $140°$. The circumference angle is $(4n - 10)°$. Find n.

Solution

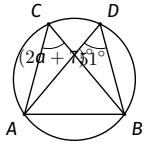
$140 = 2(4n - 10)$

$8n = 160 \Rightarrow n = 20$

Build Tasks

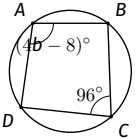
Form and solve an equation from the same-segment or cyclic-quadrilateral theorem.

9.



Find a.

10.



Find b.

11. Two angles in the same segment are $(6r - 5)^\circ$ and 73°. Find r.

12. Opposite angles in a cyclic quadrilateral are $(2s + 14)^\circ$ and $(s + 76)^\circ$. Find s.

Build Tasks — Answers

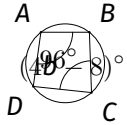
9. Find a.

Solution

$2a + 7 = 51$

$2a = 44 \Rightarrow a = 22$

10. Find b.



Solution

$4b - 8 + 96 = 180$

$4b = 92 \Rightarrow b = 23$

11. Two angles in the same segment are $(6r - 5)^\circ$ and 73°. Find r.

Solution

$6r - 5 = 73$

$6r = 78 \Rightarrow r = 13$

12. Opposite angles in a cyclic quadrilateral are $(2s + 14)^\circ$ and $(s + 76)^\circ$. Find s.

Solution

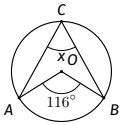
$2s + 14 + s + 76 = 180$

$3s = 90 \Rightarrow s = 30$

Push Tasks

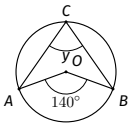
Chain the centre/circumference theorem with the base angles of isosceles triangle OAB (radii OA = OB).

1.



Find *x*, then the base angles.

2.



Find *y*, then the base angles.

3. A centre angle is $(5x - 8)^\circ$, and the same-arc circumference angle is $(2x + 10)^\circ$. Find *x*.

4. Tangent at *P*; radius *OP*; chord *PQ* with $OP = OQ$; $\angle OQP = 34^\circ$. Find the angle between the tangent and *PQ*.

Push Tasks — Answers

1. Find x, then the base angles.

Solution

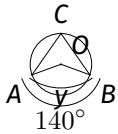
$x = 116 \div 2 = 58°$

$\triangle OAB$:

$2\times\text{base}+116 = 180$

Base angles $= 32°$ each

2. Find y, then the base angles.



Solution

$y = 140 \div 2 = 70°$

$\triangle OAB$:

$2\times\text{base}+140 = 180$

Base angles $= 20°$ each

3. A centre angle is $(5x - 8)°$, and the same-arc circumference angle is $(2x + 10)°$. Find x.

Solution

$5x - 8 = 2(2x + 10)$

$x = 28$

4. Tangent at P; radius OP; chord PQ with $OP = OQ$; $\angle OQP = 34°$. Find the angle between the tangent and PQ.

Solution

$\triangle OPQ$ isosceles: $\angle OPQ = 34°$

Tangent–radius $= 90°$

Angle $= 90 - 34 = 56°$

Push Tasks

Chain the cyclic-quadrilateral theorem with the centre/circumference theorem or the quadrilateral angle sum.

5. A cyclic quadrilateral has angles $x°, 95°, 65°, y°$ in order around the circle. Find x and y.

6. A cyclic quadrilateral has one angle $128°$. Its two adjacent angles are equal to each other. Find those two angles.

7. In a circle, centre angle $AOC = 140°$. $ABCD$ is a cyclic quadrilateral, B on the major arc. Find $\angle ABC$, then $\angle ADC$.

8. In a circle, cyclic quadrilateral $ABCD$ has $\angle ADC = 118°$, B on the major arc. Find $\angle ABC$, then centre angle $\angle AOC$.

Push Tasks — Answers

5. A cyclic quadrilateral has angles $x^\circ, 95^\circ, 65^\circ, y^\circ$ in order around the circle. Find x and y.

Solution

$x + 65 = 180 \Rightarrow x = 115$

$95 + y = 180 \Rightarrow y = 85$

6. A cyclic quadrilateral has one angle 128°. Its two adjacent angles are equal to each other. Find those two angles.

Solution

Adjacent pair is opposite each other, so they sum to 180°

Equal $\Rightarrow 90^\circ$ each

7. In a circle, centre angle $AOC = 140^\circ$. $ABCD$ is a cyclic quadrilateral, B on the major arc. Find $\angle ABC$, then $\angle ADC$.

Solution

$\angle ABC = 140 \div 2 = 70^\circ$

$\angle ADC = 180 - 70 = 110^\circ$

8. In a circle, cyclic quadrilateral $ABCD$ has $\angle ADC = 118^\circ$, B on the major arc. Find $\angle ABC$, then centre angle $\angle AOC$.

Solution

$\angle ABC = 180 - 118 = 62^\circ$

$\angle AOC = 2 \times 62 = 124^\circ$

Push Tasks

Explain and correct errors in circle-theorem reasoning.

9. A student says: "The centre angle is 150°, so the same-arc circumference angle is 85°." Explain the error and give the correct answer.

10. A student says a 100° angle can stand in a semicircle. Explain why this is impossible.

11. A cyclic quadrilateral has equal opposite angles, x° and x°. A student says this is only possible if $x = 90$. Is the student correct? Explain.

12. Explain how the tangent-radius, semicircle, and cyclic-quadrilateral theorems are all connected by right angles in circle geometry.

Push Tasks — Answers

9. A student says: "The centre angle is $150°$, so the same-arc circumference angle is $85°$." Explain the error and give the correct answer.

Solution

Circumference $=$ centre $\div 2$, not centre -65

Correct: $150 \div 2 = 75°$

10. A student says a $100°$ angle can stand in a semicircle. Explain why this is impossible.

Solution

The angle in a semicircle is always exactly $90°$ (Thales), so $100°$ is impossible

11. A cyclic quadrilateral has equal opposite angles, $x°$ and $x°$. A student says this is only possible if $x = 90$. Is the student correct? Explain.

Solution

Correct: opposite angles sum to $180°$

$x + x = 180 \Rightarrow x = 90$ always

12. Explain how the tangent-radius, semicircle, and cyclic-quadrilateral theorems are all connected by right angles in circle geometry.

Solution

Tangent–radius $= 90°$ always. A diameter's $180°$ centre angle halves to $90°$. Opposite arcs' $360°$ halves to a $180°$ angle sum.