Checking whether a triangle is right-angled

Mana Maths

Te reo Māori terms

ture a Pythagoras
Pythagoras' theorem
Open in Te Aka

tapatoru hāngai
right-angle triangle
Open in Te Aka

koki hāngai
right angle
Open in Te Aka

tāroa
hypotenuse
Open in Te Aka

Is a Triangle Right-Angled?

Key idea

A triangle is right-angled only when $a^2 + b^2 = c^2$, where c is the **longest** side and a, b are the two shorter sides.

How to check

1. Find the longest side, c.
2. Add the squares of the two shorter sides.
3. Square the longest side.
4. Equal? Then it is right-angled.

Example: yes

Check $5, 12, 13$.

$5^2 + 12^2 = 25 + 144 = 169$

$13^2 = 169$

Equal, so it **is** right-angled.

Common mistake

Do not just square the first two numbers you see. Find the **longest** side first — that is the side being tested.

When It Is Not Right-Angled

Example: no

Check $6, 8, 9$.

$6^2 + 8^2 = 36 + 64 = 100$

$9^2 = 81$

$100 \neq 81$, so it is **not** right-angled.

Scaled triples still work

Scaling a triple keeps the right angle: $6, 8, 10$ and $9, 12, 15$ are scaled $3, 4, 5$.

Common mistake

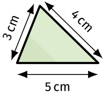
Close is not enough. Check $8, 15, 16$:

$8^2 + 15^2 = 289$ but $16^2 = 256$.

The squares must match **exactly**. Three whole-number sides do not prove a right angle.

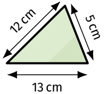
Use Pythagoras' theorem to check whether each triangle is right-angled. Look for the longest side first.

1.



Is this triangle right-angled? Write yes or no.

2.



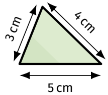
Is this triangle right-angled? Write yes or no.

3. Is a triangle with side lengths $6, 8, 10$ right-angled? Write yes or no.

4. Is a triangle with side lengths $9, 12, 16$ right-angled? Write yes or no.

Answers — compare the square of the longest side with the sum of the squares of the other two sides.

1. Is this triangle right-angled?



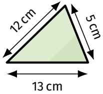
Solution

$3^2+4^2=9+16=25$

$5^2=25$, equal.

Yes.

2. Is this triangle right-angled?



Solution

$5^2+12^2=$

$25+144=169$

$13^2=169$, equal.

Yes.

3. Is a triangle with side lengths $6, 8, 10$ right-angled?

Solution

$6^2+8^2=36+64=100$

$10^2=100$, equal.

Yes.

4. Is a triangle with side lengths $9, 12, 16$ right-angled?

Solution

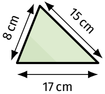
$9^2+12^2=81+144=225$

$16^2=256\neq 225.$

No.

Use Pythagoras' theorem to check whether each triangle is right-angled. Look for the longest side first.

5.



Is this triangle right-angled? Write yes or no.

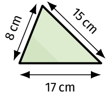
6. Is a triangle with side lengths $3, 4, 6$ right-angled? Write yes or no.

7. Is a triangle with side lengths $7, 24, 25$ right-angled? Write yes or no.

8. Is a triangle with side lengths $5, 5, 7$ right-angled? Write yes or no.

Answers — compare the square of the longest side with the sum of the squares of the other two sides.

5. Is this triangle right-angled?



Solution

$8^2 + 15^2 =$

$64 + 225 = 289$

$17^2 = 289$, equal.

Yes.

6. Is a triangle with side lengths $3, 4, 6$ right-angled?

Solution

$3^2 + 4^2 = 9 + 16 = 25$

$6^2 = 36 \neq 25.$

No.

7. Is a triangle with side lengths $7, 24, 25$ right-angled?

Solution

$7^2 + 24^2 = 49 + 576 = 625$

$25^2 = 625$, equal.

Yes.

8. Is a triangle with side lengths $5, 5, 7$ right-angled?

Solution

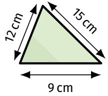
$5^2 + 5^2 = 25 + 25 = 50$

$7^2 = 49 \neq 50.$

No.

Show the square check to verify whether a triangle is right-angled, or use it to find a missing side.

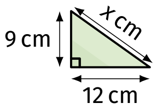
1.



Show the square check: right-angled?

2. Check whether side lengths $10, 24, 26$ make a right-angled triangle. Show the square check.

3.

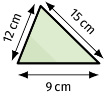


Right-angled. Find *x*.

4. Which side must be the hypotenuse for $7, 24, 25$? Then decide if the triangle is right-angled.

Answers — square check: does the sum of the two smaller squares equal the square of the longest side? For a known right angle, find the missing side.

1. Show the square check: right-angled?



Solution

$9^2 + 12^2 = 81 + 144 = 225$

$15^2 = 225$, equal.

Yes.

2. Check whether side lengths $10, 24, 26$ make a right-angled triangle.

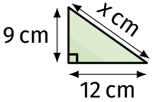
Solution

$10^2 + 24^2 = 100 + 576 = 676$

$26^2 = 676$, equal.

Yes.

3. Right-angled. Find *x*.



Solution

Legs $12, 9$; hypotenuse x.

$x^2 = 12^2 + 9^2 = 144 + 81 = 225$

$x = 15$.

4. Which side must be the hypotenuse for $7, 24, 25$? Then decide if the triangle is right-angled.

Solution

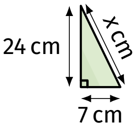
Hypotenuse $= 25$ (longest).

$7^2 + 24^2 = 49 + 576 = 625 = 25^2$.

Yes.

Show the square check to verify whether a triangle is right-angled, or use it to find a missing side.

5.

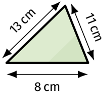


Right-angled. Find x.

6. A triangle has side lengths $9, y, 15$. If it is right-angled, find y.

7. A triangle has side lengths $12, 35, z$. If it is right-angled, find z.

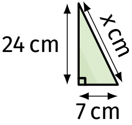
8.



Show the square check: right-angled?

Answers — square check: does the sum of the two smaller squares equal the square of the longest side? For a known right angle, find the missing side.

5. Right-angled. Find x.



Solution

Legs $7, 24$; hypotenuse x.

$x^2 = 7^2 + 24^2 = 49 + 576 = 625$

$x = 25.$

6. A triangle has side lengths $9, y, 15$. If it is right-angled, find y.

Solution

15 is the hypotenuse (longest).

$y^2 = 15^2 - 9^2 = 225 - 81 = 144$

$y = 12.$

7. A triangle has side lengths $12, 35, z$. If it is right-angled, find z.

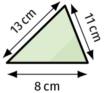
Solution

z is the hypotenuse (longest).

$z^2 = 12^2 + 35^2 = 144 + 1225 = 1369$

$z = 37.$

8. Show the square check: right-angled?



Solution

$8^2 + 11^2 = 64 + 121 = 185$

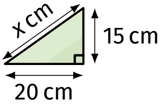
$13^2 = 169 \neq 185.$

No.

Show the square check to verify whether a triangle is right-angled, or use it to find a missing side.

9. Which side must be the hypotenuse for $20, 21, 29$? Then decide if the triangle is right-angled.

10.



Right-angled. Find *x*.

11. A student says $16, 30, 34$ is not right-angled because the numbers are too large. Is the student correct? Show the square check.

12. Sort into right-angled or not: $(9, 40, 41)$, $(8, 15, 18)$, $(11, 60, 61)$.

Answers — square check: does the sum of the two smaller squares equal the square of the longest side? For a known right angle, find the missing side.

9. Which side must be the hypotenuse for $20, 21, 29$? Then decide if the triangle is right-angled.

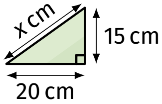
Solution

Hypotenuse $= 29$ (longest).

$20^2 + 21^2 = 400 + 441 = 841 = 29^2$.

Yes.

10. Right-angled. Find x.



Solution

Legs $20, 15$; hypotenuse x.

$x^2 = 20^2 + 15^2 = 400 + 225 = 625$

$x = 25$.

11. A student says $16, 30, 34$ is not right-angled because the numbers are too large. Is the student correct? Show the square check.

Solution

$16^2 + 30^2 = 1156 = 34^2$.

Right-angled — **student is wrong**.

12. Sort into right-angled or not: $(9, 40, 41)$, $(8, 15, 18)$, $(11, 60, 61)$.

Solution

$(9, 40, 41)$: $1681 = 41^2$, right.

$(11, 60, 61)$: $3721 = 61^2$, right.

$(8, 15, 18)$: $289 \neq 324$, not.

Apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons, justifying your reasoning.

1. A triangle has perimeter 56 and two sides 15 and $20.$ Could it be right-angled? Explain.

2. A triangle has perimeter 70 and two sides 21 and $29.$ Could it be right-angled? Explain.

3. A triangle has side lengths $3k, 4k, 5k.$ Prove it is always right-angled for positive k.

4. One side of a triangle is $25.$ Find two other whole-number sides so the triangle is right-angled, and prove your triple works.

Answers — apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons.

1. Perimeter 56, two sides 15 and 20. Could it be right-angled?

Solution

Third side $= 56 - 15 - 20 = 21.$

$15^2 + 20^2 = 225 + 400 = 625 \neq 441 = 21^2.$

No.

2. Perimeter 70, two sides 21 and 29. Could it be right-angled?

Solution

Third side $= 70 - 21 - 29 = 20.$

$20^2 + 21^2 = 400 + 441 = 841 = 29^2.$

Yes.

3. Side lengths $3k, 4k, 5k.$ Prove it is always right-angled for positive k.

Solution

$(3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2 = (5k)^2$ for every positive k.

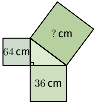
4. One side is $25.$ Find two other whole-number sides so it is right-angled, and prove your triple works.

Solution

$7, 24, 25\colon\ 7^2 + 24^2 = 625 = 25^2.$

Apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons, justifying your reasoning.

5.



Hypotenuse square: area and side length?

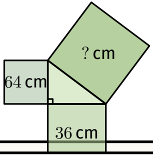
6. A student checks $9^2 + 40^2 = 41^2$ and says that proves the triangle is right-angled. What assumption did they make about the longest side?

7. Which is closer to being right-angled: $(8, 15, 16)$ or $(8, 15, 18)$? Justify using $a^2 + b^2 - c^2$.

8. Which is closer to being right-angled: $(10, 24, 25)$ or $(10, 24, 27)$? Justify using $a^2 + b^2 - c^2$.

Answers — apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons.

5. Hypotenuse square: area and side length?



Solution

$36 + 64 = 100$.
Area $= 100$, side $= 10$.

6. A student checks $9^2 + 40^2 = 41^2$ and says that proves right-angled. What assumption did they make?

Solution

That 41 is the hypotenuse (longest side).

7. Which is closer to being right-angled: $(8, 15, 16)$ or $(8, 15, 18)$? Justify using $a^2 + b^2 - c^2$.

Solution

$8^2 + 15^2 = 289$.
$289 - 16^2 = 33$; $289 - 18^2 = -35$.
$|33| < 35$, so $(8, 15, 16)$ is closer.

8. Which is closer to being right-angled: $(10, 24, 25)$ or $(10, 24, 27)$? Justify using $a^2 + b^2 - c^2$.

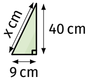
Solution

$10^2 + 24^2 = 676$.
$676 - 25^2 = 51$; $676 - 27^2 = -53$.
$|51| < 53$, so $(10, 24, 25)$ is closer.

Apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons, justifying your reasoning.

9. A triangle has sides $x, 35, 37$. Find x if the triangle is right-angled.

10.



Right-angled. Find x, and explain how you knew the hypotenuse before calculating.

11. A student says every triangle with sides in the ratio $5 : 12 : 13$ is right-angled. Are they correct? Explain using the scaling property.

12. Create a whole-number triangle close to $(9, 40, 41)$ that is not right-angled, and explain why it fails the check.

Answers — apply the converse of Pythagoras' theorem to word problems, proofs, and comparisons.

9. A triangle has sides $x, 35, 37$. Find x if the triangle is right-angled.

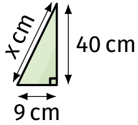
Solution

37 is the hypotenuse.

$x^2 = 37^2 - 35^2 = 1369 - 1225 = 144$

$x = 12$.

10. Right-angled. Find x; explain how you knew the hypotenuse first.



Solution

Opposite the right angle $\Rightarrow$ hyp.

$x^2 = 1681, \; x = 41$.

11. A student says every triangle with sides in the ratio $5 : 12 : 13$ is right-angled. Correct? Explain using the scaling property.

Solution

Yes — $(5k)^2 + (12k)^2 = (13k)^2$ for all k.

12. Create a whole-number triangle close to $(9, 40, 41)$ that is not right-angled, and explain why it fails.

Solution

Example: $9, 40, 42$.

$9^2 + 40^2 = 1681 \neq 1764 = 42^2$, so it fails the check.