



Applying Pythagoras' theory and trigonometry to 3D shapes

Mana Maths

Te reo Māori terms



tapatoru

triangle

Open in Te Aka

koki

angle

Open in Te Aka

pito roa

hypotenuse

Open in Te Aka

ahu-toru

3D shape

Open in Te Aka

3D Shapes: Two-Step Diagonal



Key idea

In a 3D shape, find a **face diagonal** first, then use it as one side of a second right triangle.

Method

1. Find the face diagonal.
2. Use it in the next right triangle.
3. Keep values exact till the end.

Example: space diagonal

Cuboid $6 \times 8 \times 24$.

Face: $6^2 + 8^2 = 100 \Rightarrow d = 10$.

Space: $10^2 + 24^2 = 676 \Rightarrow D = 26$.

Common mistake

Only combine edges that truly meet in one right-angled triangle — not edges from different faces.

3D Shapes: Angles and Heights



Key idea

Once the right triangle is clear, use SOH-CAH-TOA. For an angle **to the base**, the floor distance is the adjacent side.

Example: angle to the base

A line is 20 m long and rises 12 m.

$$\sin \theta = \frac{12}{20} = 0.6, \text{ so } \theta \approx 36.9^\circ.$$

Example: height from an angle

A support is 18 m long at 50° to the base.

$$\text{height} = 18 \sin 50^\circ \approx 13.8 \text{ m}.$$

Common mistake

Angle *to the base*: base is adjacent. Angle *to the vertical*: the roles switch. Read the angle carefully.



Start Tasks



1. A cuboid has base 6 cm by 8 cm and height 5 cm. Find the base diagonal.

2. A cuboid has base 9 m by 12 m and height 4 m. Find the base diagonal.

3. A box has base 5 cm by 12 cm and height 13 cm. Find the base diagonal.

4. A cuboid has base diagonal 10 cm and height 24 cm. Find the space diagonal.

5. A room has base diagonal 15 m and height 8 m. Find the space diagonal.

6. A cuboid has base diagonal 13 cm and height 84 cm. Find the space diagonal.

7. A vertical cross-section is a right triangle with angle 35° and hypotenuse 10 cm. Find the vertical height

8. A ladder inside a frame makes angle 42° with the floor and has length 9 m. Find the rise.

9. A support of length 14 cm makes angle 30° with a flat base. Find the height reached.

Start Tasks Answers



1. 10 cm

2. 15 m

3. 13 cm

4. 26 cm

5. 17 m

6. 85 cm

7. 5.7 cm

8. 6.0 m

9. 7 cm

Start Tasks



10. A cuboid is 3 cm by 4 cm by 12 cm. Find the space diagonal.

11. A cuboid is 8 cm by 15 cm by 4 cm. Find the space diagonal to 1 d.p.

12. A cuboid is 5 m by 12 m by 9 m. Find the space diagonal.

13. A sloping line across a rectangular side has length 13 cm. It makes angle 40° with the base edge. Find the height.

14. A support line is 16 m long and rises at 25° . Find the vertical rise.

15. A roof strut of length 7.5 m makes angle 52° with a horizontal beam. Find the height.

16. A cuboid has base 7 cm by 24 cm and height 10 cm. Find the space diagonal.

17. A storage box has base 20 cm by 21 cm and height 28 cm. Find the longest inside diagonal.

18. A cabinet is 1.2 m by 1.6 m by 2.0 m. Find the longest inside diagonal to 1 d.p.

Start Tasks Answers



10. 13 cm

11. 17.5 cm

12. 16 m

13. 8.4 cm

14. 6.8 m

15. 5.9 m

16. 26.9 cm

17. 40 cm

18. 2.8 m

Start Tasks



19. A cuboid has base diagonal 17 cm and height 8 cm. Find the space diagonal.

20. A cuboid has space diagonal 13 cm and height 12 cm. Find the base diagonal.

21. A cuboid has space diagonal 15 cm and base diagonal 9 cm. Find the height.

22. A line inside a box has length 18 cm and makes angle 50° with the base. Find the vertical height.

23. A line inside a prism has length 22 m and angle 33° to the base. Find the rise to 1 d.p.

24. A strut inside a frame is 11 cm long and makes angle 65° with the floor. Find the height.

25. Which theorem helps find a face diagonal in a rectangular prism?

26. Which ratio helps find height from a sloping line and an angle to the base?

27. Why might a 3D problem need two steps before you reach the final answer?

Start Tasks Answers



19. $\sqrt{353} \approx 18.8$ cm

20. 5 cm

21. 12 cm

22. 13.8 cm

23. 12.0 m

24. 10.0 cm

25. Pythagoras' theorem.

26. Sine.

27. Usually one step finds a hidden diagonal, then a second step finds the final height, angle, or distance.

Build Tasks



1. A cuboid has dimensions 8 cm, 6 cm, and 24 cm. Find the space diagonal to 1 d.p.

2. A cuboid has dimensions 10 m, 24 m, and 7 m. Find the space diagonal to 1 d.p.

3. A box is 9 cm by 12 cm by 20 cm. Find the longest inside diagonal.

4. A room is 4 m wide, 3 m high, and 12 m long. Find the distance from one floor corner to the opposite ceiling corner.

5. A crate is 5 cm by 12 cm by 84 cm. Find its longest inside diagonal.

6. A cuboid has base diagonal 15 cm and height 8 cm. Find the space diagonal.

7. A space diagonal is 18 cm and makes angle 41° with the base. Find the vertical height.

8. A support line inside a prism is 25 m and rises at 28° . Find the rise to 1 d.p.

9. A strut across a rectangular side is 14 cm long and makes angle 36° with the bottom edge. Find the height on

Build Tasks Answers



1. 25.9 cm

2. 27.0 m

3. 25 cm

4. 13 m

5. 85 cm

6. 17 cm

7. 11.8 cm

8. 11.7 m

9. 8.2 cm

Build Tasks



10. The space diagonal of a cuboid is 13 cm. Its height is 12 cm and its depth is 4 cm. Find the length.

11. The space diagonal of a box is 17 cm. Its length is 8 cm and depth is 15 cm. Find the height.

12. A cuboid is 6 cm high and has a square base. The space diagonal is 11 cm. Find the base side length to 1 d.p.

13. A tunnel frame is 9 m wide and 12 m high. A support runs across that face, then continues 20 m through the tunnel.

Find the full 3D length

14. A shipping box is 30 cm by 40 cm by 120 cm. Can a 130 cm rod fit exactly along the inside diagonal? Explain.

15. A cuboid has dimensions 2.4 m, 3.2 m, and 6 m. Find the longest inside diagonal to 1 d.p.

16. A line inside a cuboid is 26 cm long and makes angle 22° with the base. Find the vertical height.

17. A line across a rectangular face is 19 cm and makes angle 58° with the base edge. Find the vertical side on that

18. A sloping beam inside a frame is 11.5 m long and makes angle 63° with the floor. Find the rise

Build Tasks Answers



10. 3 cm

11. 0 cm, so the given numbers already make a flat 8-15-17 triangle and no extra height.

12. 5.0 cm

13. $\sqrt{9^2 + 12^2} = 15$, then
 $\sqrt{15^2 + 20^2} = 25$ m

14. No, the diagonal is
 $\sqrt{30^2 + 40^2 + 120^2} = 130$
cm, so it fits exactly rather than with spare room.

15. 7.2 m

16. 9.7 cm

17. 16.1 cm

18. 10.2 m

Build Tasks



19. A cuboid has base 12 cm by 16 cm and space diagonal 26 cm. Find the height.

20. A box has base diagonal 24 cm and space diagonal 25 cm. Find the height.

21. A line inside a prism is 30 m long and rises 12 m. Find the angle it makes with the base to 1 d.p.

22. A mast support in a 3D frame has length 18 m and height 9 m. Find the angle to the base.

23. Explain why a 3D distance problem may need Pythagoras before trigonometry.

24. Explain how you decide whether the angle is measured from the base or from the vertical side.

25. A cuboid is 9 cm by 40 cm by 41 cm. Show that its space diagonal is an integer.

26. A line inside a box is 34 cm and makes angle 45° with the base. Find the rise to 1 d.p.

27. A scaffold line is 20 m long and makes angle 37° with the floor. Find the rise and the horizontal run.

Build Tasks Answers



19. 16 cm

20. 7 cm

21. 23.6°

22. 30°

23. Because you may need a hidden face diagonal before the final triangle is known.

24. Read which line the angle is measured from; that line is the adjacent side.

25. The space diagonal is

$$\sqrt{9^2 + 40^2 + 41^2} = \sqrt{3362} = 58.$$

26. 24.0 cm

27. Rise = 12.0 m, run = 16.0 m

Push Tasks



1. A cuboid is 8 cm by 15 cm by 20 cm. Find the space diagonal exactly, then to 1 d.p.

2. A cuboid is 12 m by 16 m by 21 m. Find the longest inside diagonal to 1 d.p.

3. A tank is 9 cm by 40 cm by 41 cm. Find the space diagonal exactly if possible.

4. A 3D support has length 30 m and makes angle 37° with the floor. Find the rise and the horizontal floor distance.

5. A line inside a box is 26 cm long and rises 10 cm. Find the angle it makes with the base.

6. A prism support is 22 m long and rises 13 m. Find the angle to the base to 1 d.p.

7. A cuboid has square base side x , height 12, and space diagonal 20. Find x .

8. A cuboid has base 9 by 12 and space diagonal 17. Find the height.

9. A cuboid has height 6 and space diagonal 15. If the base is a square, find the base side.

Push Tasks Answers



1. $\sqrt{689} \approx 26.2$ cm

2. 29.0 m

3. 58 cm

4. Rise = 18.1 m, floor
distance = 24.0 m

5. 22.6°

6. 36.2°

7. $x = 8\sqrt{2} \approx 11.3$

8. 8 cm

9. $\frac{\sqrt{189}}{\sqrt{2}} \approx 9.7$

Push Tasks



10. A box is 18 cm long and 24 cm wide. Its space diagonal is 35 cm. Find the height to 1 d.p.

11. A cuboid has base diagonal 25 cm and space diagonal 39 cm. Find the height.

12. A line across a rectangular face has length 17 cm and angle 28° to the base edge. Find both face dimensions if the shorter

13. A scaffold line rises 14 m at angle 33° to the floor. Find the line length and the floor distance.

14. A line inside a prism has length 32 m and makes angle 61° with the base. Find the rise and round to 1 d.p.

15. A guy wire inside a frame is 18 m long and reaches a height of 11 m. Find the angle it makes with the ground.

16. A cuboid has dimensions x , 24, 10 and space diagonal 26. Find x .

17. A cuboid has base 20 by 21 and space diagonal 35. Show whether the height is an integer.

18. A box has base diagonal 41 cm and height 9 cm. Find the space diagonal exactly.

Push Tasks Answers



10. 19.0 cm

11. 29.9 cm

12. Opposite
 $= 17 \sin 28^\circ \approx 8.0$ cm,
adjacent
 $= 17 \cos 28^\circ \approx 15.0$ cm

13. Length ≈ 25.7 m,
floor distance ≈ 21.5 m

14. 28.0 m

15. 37.7°

16. $x = 6$

17. Height
 $= \sqrt{35^2 - 20^2 - 21^2} =$
 $\sqrt{384} = 8\sqrt{6}$, not an
integer

18. $\sqrt{1762}$ cm

Push Tasks



19. A rod fits exactly along the inside diagonal of a 12 cm by 16 cm by 21 cm box. Find the rod length.

20. Could a 50 cm rod fit inside a 9 cm by 40 cm by 41 cm box? Justify with a calculation.

21. A space diagonal is 34 cm and makes angle 45° with the base. Find the rise and compare it with the floor distance.

22. Explain why a space diagonal and two outer edges do not always form the right triangle you need first.

23. Describe a two-step method for finding a space diagonal when only the three edge lengths are known.

24. Explain how rounding too early in the face diagonal can affect the final 3D answer.

25. A cuboid is 3, 4, 12. Explain why its space diagonal is a whole number.

26. A line inside a frame has angle 30° to the base and height 9 m. Find the line length and the floor distance.

27. Write one sentence explaining how Pythagoras and trigonometry work together in 3D shape

Push Tasks Answers



19. 29 cm

20. No. The inside diagonal is 58 cm, so a 50 cm rod is too short to span it exactly.

21. Rise ≈ 24.0 cm and floor distance ≈ 24.0 cm, so they are equal.

22. The first right triangle is usually on one face; mixing unrelated edges can skip the hidden face diagonal.

23. Find the face diagonal with Pythagoras, then use that diagonal with the third edge in a second right triangle

24. Early rounding changes the diagonal you reuse, so the final 3D answer can drift.

25. Because $3^2 + 4^2 = 5^2$ and then $5^2 + 12^2 = 13^2$.

26. Length = 18 m, floor distance = $9\sqrt{3} \approx 15.6$ m

27. Pythagoras finds hidden right-triangle sides, and trigonometry then uses those sides with angles to finish the