



Algebra with geometry

Mana Maths

Te reo Māori terms



taurangi

variable

Open in Te Aka

wharite

equation

Open in Te Aka

koki

angle

Open in Te Aka

roa

length

Open in Te Aka

Algebra with Geometry - Method



Key idea



Two skills, not a new geometry fact:

1. write an equation from a geometry fact you know, then
2. solve it.

Angle and perimeter facts give a **linear** equation; area facts give a **quadratic**.

Two-step method



1. Write the geometric fact as an equation in the unknown.
2. Solve it, then check it makes sense: a length must be positive, an angle between 0° and 360° .



= **copy this into your book.** Boxes without a pencil are just to read.



Linear: Angle and Perimeter Facts



Angle on a straight line

Angles $(x + 20)^\circ$ and 70° sit on a straight line.

$$(x + 20) + 70 = 180$$

$$x + 90 = 180$$

$$x = 90$$

Rectangle perimeter

Length 6 cm, width x cm, perimeter 22 cm.

$$2(6 + x) = 22$$

$$6 + x = 11$$

$$x = 5$$

Common mistake

Pick the right angle fact first. Vertically opposite angles are **equal**; co-interior angles **sum to** 180° . The wrong fact gives the wrong equation.



= copy this into your book. Boxes without a pencil are just to read.

Quadratic: Area Facts



Rectangle area



Width x cm, length $(x + 4)$ cm,
area 60 cm^2 .

$$x(x + 4) = 60$$

$$x^2 + 4x - 60 = 0$$

$$(x + 10)(x - 6) = 0$$

$$x = 6 \text{ or } x = -10$$

Key idea



An area quadratic has two roots
but only one sensible answer.

Reject $x = -10$ — a length can-
not be negative — so $x = 6$.

Common mistake

Do not report both roots.

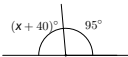
Reject the negative (or zero)
length. On an angle question,
check the final angle is positive
and under 360° .

Start Tasks



State the geometric fact as an equation, then solve for the unknown.

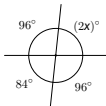
1.



Angles on a straight line are $(x + 40)^\circ$ and 95° . Find x .

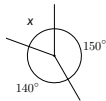
2. A right angle is split into x° and 32° . Find x .

3.



Vertically opposite angles are $(2x)^\circ$ and 84° . Find x .

4.



Angles around a point are 150° , 140° , and x° . Find x .

Start Tasks – Answers



1. Angles on a straight line are $(x + 40)^\circ$ and 95° . Find x .



Solution

$$\begin{aligned}(x + 40) + 95 &= 180 \\ x + 135 &= 180 \\ x &= 45\end{aligned}$$

3. Vertically opposite angles are $(2x)^\circ$ and 84° . Find x .



Solution

Vertically opposite:

$$\begin{aligned}2x &= 84 \\ x &= 42\end{aligned}$$

2. A right angle is split into x° and 32° . Find x .

Solution

$$\begin{aligned}x + 32 &= 90 \\ x &= 58\end{aligned}$$

4. Angles around a point are 150° , 140° , and x° . Find x .



Solution

$$\begin{aligned}150 + 140 + x &= 360 \\ 290 + x &= 360 \\ x &= 70\end{aligned}$$

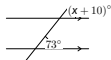
Start Tasks



State the geometric fact as an equation, then solve for the unknown.

5. Co-interior angles are x° and 108° . Find x .

6.



Corresponding angles are $(x + 10)^\circ$ and 73° . Find x .

7.



Alternate angles are $(3x)^\circ$ and 66° . Find x .

8.



A triangle has angles 55° , 65° , and x° . Find x .

Start Tasks – Answers



5. Co-interior angles are x° and 108° . Find x .

Solution

Co-interior sum to 180:

$$x + 108 = 180$$

$$x = 72$$

6. Corresponding angles are $(x + 10)^\circ$ and 73° . Find x .



Solution

Corresponding equal:

$$x + 10 = 73$$

$$x = 63$$

7. Alternate angles are $(3x)^\circ$ and 66° . Find x .



Solution

Alternate equal:

$$3x = 66$$

$$x = 22$$

8. A triangle has angles 55° , 65° , and x° . Find x .



Solution

$$55 + 65 + x = 180$$

$$120 + x = 180$$

$$x = 60$$

Start Tasks



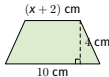
State the geometric fact as an equation, then solve for the unknown.

9.



An isosceles triangle has base angles x° and apex 40° . Find x .

11.



The trapezium has area 28 cm^2 . Find x .

10.



An equilateral triangle has each angle $(4x - 8)^\circ$. Find x .

12.

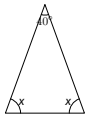


A square has side $x \text{ cm}$ and perimeter 48 cm . Find x .

Start Tasks – Answers



9. An isosceles triangle has base angles x° and apex 40° . Find x .



Solution

$$2x + 40 = 180$$

$$2x = 140$$

$$x = 70$$

10. An equilateral triangle has each angle $(4x - 8)^\circ$. Find x .



Solution

Each angle = 60° :

$$4x - 8 = 60$$

$$x = 17$$

11. The trapezium has area 28 cm^2 . Find x .



Solution

$$\frac{(x + 2) + 10}{2} \times 4 = 28$$

$$2(x + 12) = 28$$

$$x = 2$$

12. A square has side $x \text{ cm}$ and perimeter 48 cm . Find x .



Solution

$$4x = 48$$

$$x = 12$$

Start Tasks



State the geometric fact as an equation, then solve for the unknown.

13.



A rectangle has length 11 cm and width x cm. Its perimeter is 30 cm. Find x .

14.



A triangle has base 8 cm and height x cm. Its area is 32 cm^2 . Find x .

15.



A rectangle has width 6 cm and length x cm. Its area is 54 cm^2 . Find x .

16. A supplementary angle pair is x° and 112° . Find x .

Start Tasks — Answers



13. A rectangle has length 11 cm and width x cm. Its perimeter is 30 cm. Find x .



Solution

$$2(11 + x) = 30$$

$$11 + x = 15$$

$$x = 4$$

15. A rectangle has width 6 cm and length x cm. Its area is 54 cm^2 . Find x .



Solution

$$6x = 54$$

$$x = 9$$

14. A triangle has base 8 cm and height x cm. Its area is 32 cm^2 . Find x .



Solution

$$\frac{1}{2}(8)(x) = 32$$

$$4x = 32$$

$$x = 8$$

16. A supplementary angle pair is x° and 112° . Find x .

Solution

$$x + 112 = 180$$

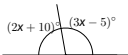
$$x = 68$$

Start Tasks



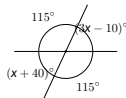
Write the geometric fact as a two-step equation, then solve for the unknown.

1.



Two angles on a straight line are $(2x + 10)^\circ$ and $(3x - 5)^\circ$. Find x .

2.



Vertically opposite angles are $(3x - 10)^\circ$ and $(x + 40)^\circ$. Find x .

3.



A triangle has angles x° , $(x + 10)^\circ$, and $(2x - 10)^\circ$. Find x .

4.

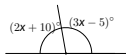


An isosceles triangle has base angles $(2x)^\circ$ and apex $(x + 50)^\circ$. Find x .

Build Tasks — Answers



1. Two angles on a straight line are $(2x + 10)^\circ$ and $(3x - 5)^\circ$. Find x .



Solution

$$\begin{aligned}(2x + 10) + (3x - 5) &= 180 \\ 5x + 5 &= 180 \\ x &= 35\end{aligned}$$

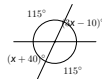
3. A triangle has angles x° , $(x + 10)^\circ$, and $(2x - 10)^\circ$. Find x .



Solution

$$\begin{aligned}x + (x + 10) + (2x - 10) &= 180 \\ 4x &= 180 \\ x &= 45\end{aligned}$$

2. Vertically opposite angles are $(3x - 10)^\circ$ and $(x + 40)^\circ$. Find x .



Solution

Vertically opposite:

$$\begin{aligned}3x - 10 &= x + 40 \\ 2x &= 50 \\ x &= 25\end{aligned}$$

4. An isosceles triangle has base angles $(2x)^\circ$ and apex $(x + 50)^\circ$. Find x .



Solution

$$\begin{aligned}2(2x) + (x + 50) &= 180 \\ 5x + 50 &= 180 \\ x &= 26\end{aligned}$$

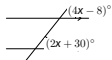
Start Tasks



Write the geometric fact as a two-step equation, then solve for the unknown.

5. Co-interior angles are $(2x + 20)^\circ$ and $(x + 40)^\circ$. Find x .

6.



Corresponding angles are $(4x - 8)^\circ$ and $(2x + 30)^\circ$. Find x .

7.



A regular hexagon has interior angle $(3x + 30)^\circ$. Find x .

8.



A rectangle has width x cm and length $(x + 4)$ cm. Its perimeter is 40 cm. Find x .

Build Tasks — Answers



5. Co-interior angles are $(2x + 20)^\circ$ and $(x + 40)^\circ$. Find x .

Solution

Co-interior sum to 180:

$$(2x + 20) + (x + 40) = 180$$

$$3x + 60 = 180, x = 40$$

6. Corresponding angles are $(4x - 8)^\circ$ and $(2x + 30)^\circ$. Find x .



Solution

Corresponding equal:

$$4x - 8 = 2x + 30$$

$$2x = 38, x = 19$$

7. A regular hexagon has interior angle $(3x + 30)^\circ$. Find x .



Solution

Hexagon interior = 120:

$$3x + 30 = 120$$

$$x = 30$$

8. A rectangle has width x cm and length $(x + 4)$ cm. Its perimeter is 40 cm. Find x .



Solution

$$2[(x + 4) + x] = 40$$

$$4x + 8 = 40$$

$$x = 8$$

Start Tasks



Write the geometric fact as a two-step equation, then solve for the unknown.

9.



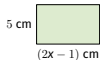
A square has side $(x + 3)$ cm and perimeter 72 cm. Find x .

10.



A triangle has base $(x + 2)$ cm and height 10 cm. Its area is 60 cm^2 . Find x .

11.



A rectangle has width 5 cm and length $(2x - 1)$ cm. Its area is 45 cm^2 . Find x .

12.



Similar rectangles have corresponding sides 6, 9 and x , 12. Find x .

Build Tasks — Answers



- 9.** A square has side $(x + 3)$ cm and perimeter 72 cm. Find x .



Solution

$$4(x + 3) = 72$$

$$x + 3 = 18$$

$$x = 15$$

- 10.** A triangle has base $(x + 2)$ cm and height 10 cm. Its area is 60 cm^2 . Find x .



Solution

$$\frac{1}{2}(x + 2)(10) = 60$$

$$x + 2 = 12$$

$$x = 10$$

- 11.** A rectangle has width 5 cm and length $(2x - 1)$ cm. Its area is 45 cm^2 . Find x .



Solution

$$5(2x - 1) = 45$$

$$2x - 1 = 9$$

$$x = 5$$

- 12.** Similar rectangles have corresponding sides 6, 9 and x , 12. Find x .



Solution

$$\frac{6}{9} = \frac{x}{12}$$

$$9x = 72$$

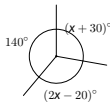
$$x = 8$$

Start Tasks



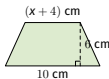
Write the geometric fact as a two-step equation, then solve for the unknown.

13.



Angles around a point are $(x + 30)^\circ$, $(2x - 20)^\circ$, and 140° . Find x .

15.



The trapezium has area 48 cm^2 . Find x .

14.



Alternate angles are $(3x - 15)^\circ$ and $(x + 45)^\circ$. Find x .

16.



A regular nonagon (9 sides) has interior angle $(2x + 20)^\circ$. Find x .

Build Tasks — Answers



13. Angles around a point are $(x + 30)^\circ$, $(2x - 20)^\circ$, and 140° . Find x .



Solution

$$(x + 30) + (2x - 20) + 140 = 360$$

$$3x + 150 = 360$$

$$x = 70$$

14. Alternate angles are $(3x - 15)^\circ$ and $(x + 45)^\circ$. Find x .



Solution

Alternate equal:

$$3x - 15 = x + 45$$

$$2x = 60, x = 30$$

15. The trapezium has area 48 cm^2 . Find x .



Solution

$$\frac{(x + 4) + 10}{2} \times 6 = 48$$

$$3(x + 14) = 48$$

$$x = 2$$

16. A regular nonagon (9 sides) has interior angle $(2x + 20)^\circ$. Find x .



Solution

Nonagon interior = 140:

$$2x + 20 = 140$$

$$x = 60$$

Start Tasks



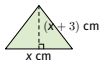
Write the geometric fact as an equation. If it is quadratic, find both roots and reject any that cannot be a length.

1.



A rectangle has width $x \text{ cm}$ and length $(x + 4) \text{ cm}$. Its area is 60 cm^2 . Find x .

2.



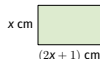
A triangle has base $x \text{ cm}$ and height $(x + 3) \text{ cm}$. Its area is 44 cm^2 . Find x .

3.



A rectangle has width $x \text{ cm}$ and length $(x - 2) \text{ cm}$. Its area is 63 cm^2 . Find x .

4.



A rectangle has width $x \text{ cm}$ and length $(2x + 1) \text{ cm}$. Its area is 55 cm^2 . Find x .

Push Tasks — Answers



1. A rectangle has width x cm and length $(x + 4)$ cm. Its area is 60 cm^2 . Find x .



Solution

$$\begin{aligned}x(x + 4) &= 60 \\x^2 + 4x - 60 &= 0 \\(x + 10)(x - 6) &= 0 \\x &= 6 \text{ (reject } -10)\end{aligned}$$

2. A triangle has base x cm and height $(x + 3)$ cm. Its area is 44 cm^2 . Find x .



Solution

$$\begin{aligned}\frac{1}{2}x(x + 3) &= 44 \\x^2 + 3x - 88 &= 0 \\(x + 11)(x - 8) &= 0 \\x &= 8 \text{ (reject } -11)\end{aligned}$$

3. A rectangle has width x cm and length $(x - 2)$ cm. Its area is 63 cm^2 . Find x .



Solution

$$\begin{aligned}x(x - 2) &= 63 \\x^2 - 2x - 63 &= 0 \\(x - 9)(x + 7) &= 0 \\x &= 9 \text{ (reject } -7)\end{aligned}$$

4. A rectangle has width x cm and length $(2x + 1)$ cm. Its area is 55 cm^2 . Find x .



Solution

$$\begin{aligned}x(2x + 1) &= 55 \\2x^2 + x - 55 &= 0 \\(2x + 11)(x - 5) &= 0 \\x &= 5 \text{ (reject } -5.5)\end{aligned}$$

Start Tasks



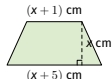
Write the geometric fact as an equation. If it is quadratic, find both roots and reject any that cannot be a length.

5.



Similar triangles have corresponding sides 5, 8 and x , $(x + 9)$. Find x .

6.



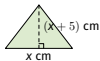
The trapezium has area 54 cm^2 . Find x , rejecting any root that cannot be a length.

7.



A rectangle has width $x \text{ cm}$ and length $(x + 3) \text{ cm}$. Its area is 70 cm^2 . Find x , then find the perimeter.

8.

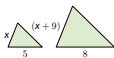


A triangle has base $x \text{ cm}$ and height $(x + 5) \text{ cm}$. Its area is 52 cm^2 . Find x , then find the height.

Push Tasks — Answers



5. Similar triangles have corresponding sides 5, 8 and x , $(x + 9)$. Find x .



Solution

$$\begin{aligned}\frac{5}{8} &= \frac{x}{x+9} \\ 5(x+9) &= 8x \\ 3x &= 45 \\ x &= 15\end{aligned}$$

7. A rectangle has width x cm and length $(x + 3)$ cm. Its area is 70 cm^2 . Find x , then find the perimeter.



Solution

$$\begin{aligned}x(x+3) &= 70 \\ (x+10)(x-7) &= 0 \\ x &= 7; \text{ perimeter} = 34 \text{ cm}\end{aligned}$$

6. The trapezium has area 54 cm^2 . Find x , rejecting any root that cannot be a length.



Solution

$$\begin{aligned}\frac{(x+1) + (x+5)}{2} \times x &= 54 \\ (x+3)x &= 54 \\ x^2 + 3x - 54 &= 0 \\ (x+9)(x-6) &= 0 \\ x &= 6 \text{ (reject } -9)\end{aligned}$$

8. A triangle has base x cm and height $(x + 5)$ cm. Its area is 52 cm^2 . Find x , then find the height.



Solution

$$\begin{aligned}\frac{1}{2}x(x+5) &= 52 \\ x^2 + 5x - 104 &= 0 \\ (x+13)(x-8) &= 0 \\ x &= 8; \text{ height} = 13 \text{ cm}\end{aligned}$$

Start Tasks



Write the geometric fact as an equation. If it is quadratic, find both roots and reject any that cannot be a length.

9.



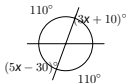
A square has side length $(x + 1)$ cm and area 100 cm^2 . Find x .

10.



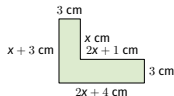
A triangle has angles $(x + 5)^\circ$, $(2x - 15)^\circ$, $(3x + 10)^\circ$. Find x and the largest angle.

11.



One angle is $(3x + 10)^\circ$, vertical opposite $(5x - 30)^\circ$. Find x and an adjacent angle.

12.



The L-shape has area 75 cm^2 . Find x , then find the perimeter.

Push Tasks — Answers



- 9.** A square has side length $(x + 1)$ cm and area 100 cm^2 . Find x .



Solution

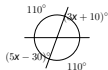
$$(x + 1)^2 = 100$$

$$x + 1 = 10$$

$$x = 9$$

(reject $x + 1 = -10$)

- 11.** One angle is $(3x + 10)^\circ$, vertical opposite $(5x - 30)^\circ$. Find x and an adjacent angle.



Solution

Vertically opposite:

$$3x + 10 = 5x - 30$$

$$2x = 40, x = 20$$

adjacent angle $= 110^\circ$

- 10.** A triangle has angles $(x + 5)^\circ$, $(2x - 15)^\circ$, $(3x + 10)^\circ$. Find x and the largest angle.



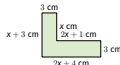
Solution

$$(x + 5) + (2x - 15) + (3x + 10) = 180$$

$$6x = 180, x = 30$$

largest angle $= 100^\circ$

- 12.** The L-shape has area 75 cm^2 . Find x , then find the perimeter.



Solution

$$3(x + 3) + 3(2x + 1) = 75$$

$$9x + 12 = 75, x = 7$$

Perimeter = sum of 6 sides

$$= 10 + 3 + 7 + 15 + 3 + 18 = 56 \text{ cm}$$

Start Tasks



Write the geometric fact as an equation. If it is quadratic, find both roots and reject any that cannot be a length.

13.



A rectangle has width x cm and length $(x + 7)$ cm. Its area is 120 cm^2 . Find x .

15. A regular polygon has interior angle $(8x + 24)^\circ$ and exterior angle $(2x + 6)^\circ$ at each vertex. Find x , then state how many sides the polygon has.

14.



A triangle has base $(x + 6)$ cm and height x cm. Its area is 56 cm^2 . Find x .

16. An isosceles triangle has two equal sides of length x cm. Its base is $(x - 2)$ cm and its perimeter is 34 cm. Find x .

Push Tasks — Answers



13. A rectangle has width x cm and length $(x + 7)$ cm. Its area is 120 cm^2 . Find x .



Solution

$$x(x + 7) = 120$$

$$x^2 + 7x - 120 = 0$$

$$(x + 15)(x - 8) = 0$$

$$x = 8 \text{ (reject } -15)$$

14. A triangle has base $(x + 6)$ cm and height x cm. Its area is 56 cm^2 . Find x .



Solution

$$\frac{1}{2}(x + 6)x = 56$$

$$x^2 + 6x - 112 = 0$$

$$(x + 14)(x - 8) = 0$$

$$x = 8 \text{ (reject } -14)$$

15. A regular polygon has interior angle $(8x + 24)^\circ$ and exterior angle $(2x + 6)^\circ$ at each vertex. Find x , then state how many sides the polygon has.

Solution

$$(8x + 24) + (2x + 6) = 180$$

$$10x + 30 = 180, x = 15$$

$$\text{exterior} = 36^\circ$$

$$n = 360 \div 36 = 10 \text{ sides}$$

16. An isosceles triangle has two equal sides of length x cm. Its base is $(x - 2)$ cm and its perimeter is 34 cm. Find x .

Solution

$$x + x + (x - 2) = 34$$

$$3x - 2 = 34$$

$$3x = 36$$

$$x = 12$$